Development of Predicted Paths for ACT Aspire Score Reports

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Abstract

This report focuses on the initial development of the predicted score paths for ACT Aspire reporting. The paths provide predicted score ranges for the next two years – as well as predicted ACT score ranges for tests administered at grades 9 and 10. Longitudinal ACT Aspire data for students tested in spring 2013 and spring 2014, as well as historical data from ACT's college and career readiness assessment system, are used to develop the predicted paths. Coverage rates of the predicted paths are examined, and proposed uses and limitations of the predicted paths are discussed.

Key words: ACT Aspire, Predicted Paths, Growth Model, Longitudinal Data, Quantile Regression

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Introduction

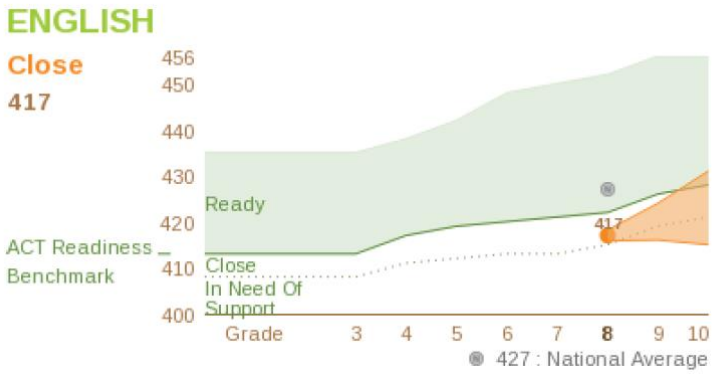
ACT Aspire will report student scores over multiple years, providing longitudinal progress reporting from grades 3-10. Score reports will include:

- A student's current and prior year scores in all tested subjects (English, Mathematics, Reading, Science, and Writing)
- Comparison to ACT Readiness Benchmarks that indicate whether students are on target to meet ACT's College Readiness Benchmarks in spring of grade 11
- Predicted score paths that provide ranges for a student's expected scores in future years
- Predicted ACT score ranges (for grades 9 and 10)
- Classification of student growth as "low", "average", or "high" based on student growth percentiles

This report focuses on the development of the predicted score paths – which provide predicted score ranges for the next two years – as well as the predicted ACT score ranges. A prototype report that illustrates the predicted path is given in Figure 1. The predicted path is represented by a cone-shaped orange-shaded area that covers two years. In this example, an eighth grade student scored 417 on the ACT Aspire English Assessment. Her predicted path covers the score range 416-424 for ninth grade and 415-431 for tenth grade.

Figure 1

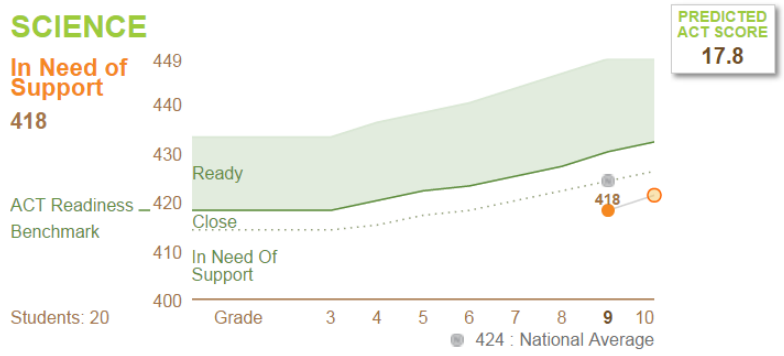
ACT Aspire Prototype Student Progress Report



In addition to the student reports, predicted paths will also be provided for classroom, school, district, and state aggregate progress reports. For aggregate reports, students are grouped according to their current year classroom, school, or district. The predicted paths will only cover one year and will show the group's predicted mean score (see Figure 2 for an excerpt from an example aggregate report). The aggregate predicted paths are represented by a line segment with the current year mean score as one endpoint (orange-filled circle) and the next year's predicted mean score as the other endpoint (orange-outlined circle). For grades 9 and 10, ACT Aspire will also report predicted mean ACT scores, as shown in Figure 2.

Figure 2

ACT Aspire Prototype District Progress Report



Predicted paths will be reported to enhance understanding of where a student (or a group of students) is likely to score in future years, assuming typical growth. This information can be used to:

- Determine if students are likely to meet ACT Readiness Benchmarks over the next two years
- Determine which students are unlikely to meet a future-year standard and thus are candidates for extra academic support
- Predict aggregate future achievement for a classroom, school, district, or state
- Predict ACT score ranges (for grades 9 and 10)

In this report, the procedures used to develop the predicted paths are documented and some statistical properties of the predicted paths are examined.

Methods

Longitudinal Samples

The ACT Aspire summative assessments were first administered in spring 2013 as part of special studies. In spring 2014, ACT Aspire summative assessments were launched. The sample

of students who tested in spring 2013 overlaps with those tested in spring 2014. Moreover, many of the 10th grade students who tested in spring 2013 took the ACT Assessment in spring 2014. The students who tested in 2013 and 2014 form a longitudinal sample of ACT Aspire-tested students. The ACT Aspire-tested longitudinal sample sizes are given in Table 1 for each subject and grade level pair for which predicted paths will be developed.

Table 1

ACT Aspire-tested longitudinal sample sizes

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing	Composite
3-4	3,784	8,843	8,943	3,403	469	
4-5	3,912	8,724	8,694	3,070	487	
5-6	3,563	7,982	8,008	3,441	431	
6-7	2,329	5,876	5,880	1,677	381	
7-8	1,924	4,583	4,579	2,132	262	
8-9	241	261	118	156	116	34
9-10	26	21	20	21	0	15
9-11 (ACT)	0	0	0	0	0	0
10-11 (ACT)	3,992	3,774	3,540	3,525	922	2,851

Ideally, the development of predicted score ranges would be based on large longitudinal samples of ACT Aspire-tested students. From Table 1, we see that the sample sizes vary considerably across grade levels and subject areas. In spring 2014, the state of Alabama administered ACT Aspire Mathematics and Reading to all students in grades 3-8. Many districts in Alabama also administered English, Science, and Writing to grade 3-8 students. Thus, the longitudinal sample sizes are larger for the pre-high school grade level pairs and considerably smaller for grade level pairs 8-9 and 9-10. Moreover, there are relatively few students who took ACT Aspire Writing tests in both 2013 and 2014. Because students have been assessed for at most two years as of spring 2014, there are no students who have taken ACT Aspire in grade 9

and the ACT in grade 11. For grades 3-7, ACT Aspire Composite scores will not be reported and so sample sizes are not given.

ACT has developed a concordance table for estimating ACT Aspire scores from ACT Explore or ACT Plan scores (http://www.act.org/migrate/pdf/ConcordanceTable.pdf). In recent years, there are large samples of students who tested in grades 8-11 with ACT Explore, ACT Plan, and/or the ACT. After using the concordance tables to estimate ACT Aspire scores from ACT Explore or ACT Plan scores, these samples can be used to generate concordance-derived longitudinal samples of ACT Aspire scores. This approach was applied to large samples of students who completed high school in 2014. The one-year grade level pairs include students who tested 10-14 months apart; the 9-11 grade level pair includes students who tested 22-26 months apart. The concordance-derived longitudinal sample sizes are given in Table 2 for each subject and grade level pair.

Table 2

Concordance-derived longitudinal samples of ACT Aspire scores

Grade Level Pair	Test 1	Test 2	Subject Area				
			English	Mathematics	Reading	Science	Composite
8-9	ACT Explore	ACT Explore	53,537	53,537	53,537	53,537	53,537
9-10	ACT Explore	ACT Plan	172,339	172,339	172,339	172,339	172,339
9-11	ACT Explore	ACT	50,656	50,656	50,656	50,656	50,656
10-11	ACT Plan	ACT	148,356	148,356	148,356	148,356	148,356

Note that Writing scores are not represented in the concordance-derived samples because neither ACT Explore nor ACT Plan includes a Writing test. Also note that the sample sizes are constant across subject areas because the Explore, Plan, and the ACT test batteries are

administered with all four subjects and students with missing test scores were deleted from the analysis file.

Statistical Methods

Given the available data and the intended uses of the predicted paths, we chose to base the one-year predicted paths on the estimated 25th and 75th percentile of the test score distribution, conditional on the prior year test score. Using an inclusive definition for predicted path coverage (scores greater than or equal to the lower score of the projected range and scores less than or equal to the upper score of the range), more than 50% of test scores would be expected to fall within the one-year predicted path. Less than 25% of test scores would be above the one-year predicted path score range and less than 25% would be below.

The full predicted path, which encompasses the one-year and two-year predictions (see Figure 1), is drawn by extending the one-year predictions for another year in a linear fashion. Let C_x represent the current year score at grade g, and let P_{25} and P_{75} represent the 25th and 75th percentiles of the distribution of grade g+1 scores, conditional on a grade g test score of C_x. The predicted path is thus drawn as a triangle with vertices (g, C_x), (g+2, $2P_{25}$- C_x,), (g+2, $2P_{75}$- C_x).

One-year predicted mean scores will be used to form aggregate predicted paths, and the aggregate predicted paths will be drawn as line segments as opposed to triangles (see Figure 2). Individual student predicted scores are based on the estimated 50th percentile of the test score distribution (P_{50}), conditional on the prior year test score (C_x). Predicted mean scores for classrooms, schools, and districts are calculated as the mean of individual student predicted scores. Unlike the predicted paths for student score reports, the aggregate paths will not convey information about the uncertainty of the prediction. For classrooms, schools, and districts, two primary sources of uncertainty in predicted scores are 1) variation across units in expected

growth, and 2) sampling error within units that is generally larger with smaller sample size. Future research may attempt to develop predicted paths for aggregates that convey uncertainty.

To estimate P_{25}, P_{50}, and P_{75}, quantile regression (Koenker & Bassett, 1978; Koenker, 2005) was used. Quantile regression is conceptually similar to ordinary least-squares regression, which is used to estimate the mean of an outcome (Y) given a set of predictor variables (denoted X). Quantile regression estimates selected quantiles of an outcome or dependent variable (denoted Y), given a set of predictor variables (denoted X). The quantile regression models were estimated using the QUANTREG procedure in SAS (Chen, 2005).

For our purposes, quantile regression was used to estimate selected quantiles of spring 2014 test scores (denoted Y), given specific values of spring 2013 test scores (denoted X). The desired quantiles (0.25, 0.50, and 0.75 corresponding to the 25^{th}, 50^{th}, and 75^{th} percentiles) are specified in the SAS QUANTREG procedure.

For all subject areas except Writing, separate regression models were fit for each grade level pair and each subject area. For three of the grade level pairs (8-9, 9-10, 9-11), the concordance-derived longitudinal samples were used because the sample sizes for the ACT Aspire-tested longitudinal samples (Table 1) were too small. For the 10-11 grade level pair, the Aspire-ACT matched data sets were used for all subjects (including Writing) and the Composite score.

For Writing, a single model was fit that combined data from all grade level pairs (3-4 through 8-9, yielding a total sample size of 2,146). To allow for potential grade level effects, grade-level dummy variables were also included in the model, resulting in grade level pair-specific intercepts. No longitudinal Writing data exist for grade 9-10, so it is assumed that the quantile regression equations for grade level pairs 8-9 and 9-10 are equal. Because no data are

available to estimate predicted ACT Writing scores for grade 9 ACT Aspire Writing scores, the predicted ACT Writing scores for grade 10 Writing scores are also used for grade 9.

Table 3

Longitudinal samples used to develop predicted paths using quantile regression

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing[1]	Composite
3-4	3,784	8,843	8,943	3,403	469	
4-5	3,912	8,724	8,694	3,070	487	
5-6	3,563	7,982	8,008	3,441	431	
6-7	2,329	5,876	5,880	1,677	381	
7-8	1,924	4,583	4,579	2,132	262	
8-9	*53,537	*53,537	*53,537	*53,537	116	*53,537
9-10	*172,339	*172,339	*172,339	*172,339	0	*172,339
9-11 (ACT)	*50,656	*50,656	*50,656	*50,656	0	*50,656
10-11 (ACT)	3,992	3,774	3,540	3,525	922	2,851

*Indicates concordance-derived sample

Evaluating the Results

Checks were done to examine the logical consistency of the predicted paths. The following propositions were examined:

1) For scores that are at the ACT Readiness Benchmark, the predicted paths should include the ACT Readiness Benchmarks for the next two grade levels. Given the interpretation of the ACT Readiness Benchmarks as the scores needed to be on target for college and career readiness by spring grade 11, it is expected that students who meet a Benchmark should have a reasonable chance of also meeting the Benchmark the next two years.
2) The one-year predicted path values (lower, upper, and predicted value) should be monotonically increasing across the score scale within each grade level and subject.

[1] For grade level pairs 3-4 through 8-9, data were combined for fitting the quantile regression models.

Moreover, for grades 9 and 10, predicted ACT scores (lower, upper, and predicted value) should be monotonically increasing across the score scale within each grade level and subject.

3) For each grade 9 score, the predicted ACT score range should be at least as wide as the predicted ACT score ranges for the same grade 10 score. Because grade 10 is in closer proximity to the grade 11 ACT, there should be less variability in predicted ACT scores for grade 10 scores relative to grade 9 scores.
4) For each grade 9 score, the predicted ACT score values should be greater than the predicted ACT score values for the same grade 10 score. Because grade 10 is in closer proximity to the grade 11 ACT, there is less time for academic growth to occur, and so the predicted ACT scores should be smaller for grade 10 scores relative to those for grade 9 scores.

If any of the consistency propositions were not met, but were nearly met, hand-adjustments would be made to the predicted paths to correct for the inconsistency. If any of the consistency propositions failed considerably, the methodology would need to be revisited.

Following the consistency checks and any hand adjustments, descriptive statistics were examined including score means and standard deviations, correlations, gain score means and standard deviations, and proportion of examinees with positive gain scores. For all subject areas except Writing, ACT Aspire scores share a common scale across grade levels and positive mean gain scores are anticipated because students are expected to increase their knowledge and skills in the tested areas after one year of schooling. The ACT Aspire Writing tests have four domains, each of which is scored using a rubric with a score of "4" indicating expected grade level

performance. The domain scores determine a total raw score for the Writing test, which is then converted to a scale score. The scaling and scoring process for Writing implies that one should not necessarily expect positive gain scores, and that gain scores do not have a clear meaning for Writing. Rather, consistency in ACT Aspire Writing scores across grade levels suggests the same performance relative to grade level expectations.

Following the descriptive statistics, coverage properties of the predicted paths were examined, including:

1) The percentage of students scoring below, within, and above one-year ACT Aspire predicted paths. For this analysis, the ACT Aspire longitudinal sample was used (sample sizes given in Table 1) to evaluate the predicted paths. For grade level pairs 3-4 through 7-8, and 10-11, the samples are used both for developing the predicted paths and for evaluating the predicted paths. Ideally, the development and evaluation samples would be independent. However, given that only one year of longitudinal data are available with limited sample sizes, this was not feasible. For grade level pairs 8-9, 9-10, and 9-11, the concordance-derived longitudinal sample was used to develop the predicted paths and the ACT Aspire longitudinal sample was used to evaluate the predicted paths. Thus, the development and evaluation samples are independent. However, for grade level pairs 8-9 and 9-10, the evaluation samples are small.
2) The percentage of students scoring below, within, and above one-year and two-year predicted paths derived using EPAS data (EPAS is the former name of ACT's longitudinal assessment system that features ACT Explore, ACT Plan, and the ACT). This analysis did not use ACT Aspire data, but rather relied on recent EPAS data. The same procedure used to develop predicted paths for ACT Aspire was applied to the EPAS

data. Because multi-year longitudinal samples are available, this data allow us to study the properties of two-year predicted paths. While not directly applicable to ACT Aspire, the results should provide a general understanding of the properties of predicted paths that are developed using the stated procedures. Students who completed high school in 2013 were used to develop the predicted paths and students who completed high school in 2014 were used to evaluate the predicted paths. This analysis was done for grade level pairs 8-9, 9-10, 10-11, 8-10, and 9-11.

3) The percentage of students whose predicted path would predict strictly positive or strictly negative score gain. ACT Aspire features a vertical scale and positive gain scores (the arithmetic difference of the current year score and prior year score) suggest growth. While positive gain scores are more likely to occur and are consistent with the generally held idea that K-12 students gain knowledge and skills after one year of schooling, a combination of factors can lead to negative gain scores, including:
    a. Test measurement error. A student who scores above his true score in grade *k* and below his true score in grade *k*+1 is more likely to see a decline in his test score.
    b. No change or a decline in the knowledge and skills tested during the time between tests.

Therefore, it may be reasonable for predicted score paths to include negative gain scores. For this analysis, we used ACT Aspire score results from spring 2013 and assigned predicted paths to all scores. We then classified each prediction as A) *strictly positive* if the one-year predicted path only included positive gain scores, B) *strictly negative* if the one-year predicted path only included negative gain scores, and C) *other* if the predicted path included both positive and negative gain scores or if the predicted path included gain scores of 0.

Results

Logical Consistency Checks

Two of the four logical consistency checks were met with no exceptions: 1) For scores that are at the ACT Readiness Benchmark, all of the predicted paths included the ACT Readiness Benchmarks for the next two grade levels and 2) The one-year predicted path values (lower, upper, and predicted value) were all monotonically increasing across the score scale within each grade level and subject and, for grades 9 and 10, predicted ACT scores (lower, upper, and predicted value) were all monotonically increasing across the score scale within each grade level and subject.

The other two consistency checks dealt with logical agreement between ACT score predictions for grade 9 and 10 scores. For each grade 9 score, the predicted ACT score range should be at least as wide as the predicted ACT score ranges for the same grade 10 score. And, for each grade 9 score, the predicted ACT score values should be greater than the predicted ACT score values for the same grade 10 score. These two propositions were not met in all cases. However, the deviations from the propositions were minor and so hand adjustments were applied to correct the predicted values. The ACT score predictions for grade 10 were developed using samples of students tested with ACT Aspire and the ACT, whereas the score predictions for grade 9 were developed using the concordance-derived samples. Thus, the grade 10 predictions were deemed more reliable and the hand adjustments were made to the grade 9 values. The hand adjustment process involved widening some of the grade 9 predicted ACT score ranges (usually by one score point) and increasing some of the grade 9 predicted ACT score values. The vast majority of hand adjustments to the predicted ACT score ranges were made at the extremes of

the score distributions (very low scores and very high scores). All of the hand adjustments to the predicted ACT score values were made in the lower extremes of the score distributions.

Descriptive Statistics

The gender and racial/ethnic breakdowns of the longitudinal samples are given in Table 4. Male and female students are evenly represented, with the largest imbalance observed for grade 10-11 (51.8% female, 46.7% male). The early grade level pairs (3-4 through 7-8) are predominantly White (57.5%-62.1%) or Black/African American (25.8% to 32.0%). The percentage of students of Hispanic ethnicity ranged from 4.7% to 6.0% for grades 3-4 through 7-8, but was much higher in the concordance-derived samples (15.3% to 27.2%). The share of Black/African American students was smaller in the concordance-derived samples (8.8% to 16.6%), but was larger in the grade 10-11 sample (25.6%). Across grade levels, there were small percentages of students from the other racial/ethnic groups (American Indian/Alaska Native, Asian, Native Hawaiian/Pacific Islander, and two or more races).

Table 4

Gender and race/ethnicity of students in longitudinal samples

Group	Grade Level Pair								
	3-4	4-5	5-6	6-7	7-8	8-9	9-10	9-11	10-11
Gender									
Female	48.7	49.1	49.9	48.1	50.0	50.2	49.7	50.6	51.8
Male	51.2	50.7	49.7	51.8	49.8	49.8	50.2	49.4	46.7
Missing	0.1	0.2	0.4	0.0	0.2	0.0	0.0	0.0	1.5
Race/ethnicity									
American Indian/Alaska Native	1.5	1.5	1.9	1.8	1.5	0.5	0.5	0.4	0.2
Asian	2.1	1.9	1.4	1.2	1.3	4.0	4.0	3.4	2.7
Black/African American	27.1	25.8	27.9	28.6	32.0	16.6	12.3	8.8	25.6
Hispanic/Latino	6.0	5.2	5.1	4.8	4.7	27.2	20.2	15.3	7.9
Native Hawaiian/Pacific Islander	0.1	0.1	0.3	0.1	0.0	0.2	0.3	0.3	0.2
Two or more races	0.0	0.0	0.0	0.0	0.0	3.8	4.2	4.5	4.1
White	59.7	62.1	59.3	61.0	57.5	47.2	57.1	66.5	53.2
Missing	3.6	3.4	4.1	2.5	3.0	0.6	1.5	0.8	6.1

The longitudinal samples for the early grade levels (3-4 through 7-8) come almost exclusively from the East region[2] of the United States (Table 5), owing to Alabama's adoption of ACT Aspire for grades 3-8 in spring 2014. The concordance-derived samples (grades 8-9, 9-10, and 9-11) come mostly from the Midwest (67.1% to 82.5%) and West (13.7% to 22.3%). The grade 10 to 11 (Aspire to ACT) sample comes mostly from the East (55.8%) and Midwest (34.59%).

Table 5

Regions represented in longitudinal samples

Region	Grade Level Pair								
	3-4	4-5	5-6	6-7	7-8	8-9	9-10	9-11	10-11
East	99.7	99.1	98.2	99.5	99.0	1.9	5.9	2.4	55.8
Midwest	0.3	0.4	0.5	0.5	1.0	76.3	67.1	82.5	34.5
Southwest			0.1	<0.1		3.7	4.8	1.4	6.3
West		0.5	1.2			18.1	22.3	13.7	3.5

Descriptive statistics of the test scores are provided in Tables 6-9. The prior score means and standard deviations (Table 6) are used to assess the achievement distribution of the samples used to develop the predicted paths. Estimates of national mean scores[3], which will be included on ACT Aspire score reports, are presented below the sample mean and standard deviation. In most cases, the sample mean matches the national mean within one score point. The grade 8 mean scores for the grade 8-9 sample are smaller than the national means for English (424.3 vs. 427), Reading (418.3 vs. 422), and Science (420.8 vs. 423), but are higher for Writing (426.7 vs.

[2] Note that the region definitions used by ACT and used in this report are different than those used by the U.S. Census Bureau, with large shares of states assigned to the East region (20 states plus the District of Columbia) and West region (16 states).

[3] The national mean scores were estimated using ACT Aspire test scores from 2013 and 2014 for students who took the on grade test (e.g., 5th grade students who took the 6th grade test were excluded). The sample was not reweighted to match national demographics.

423). The grade 10 mean scores for the grade 10-11 sample are smaller than the national means for all subject areas, with Reading and Science having the largest differences.

Table 6

Prior Score Means and Standard Deviations, National Means

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing	Composite
3-4	416.6 (5.8) 416	412.2 (3.9) 412	412.3 (5.2) 412	414.1 (6.0) 414	423.5 (5.7) 421	
4-5	420.0 (6.0) 420	414.9 (3.9) 415	414.6 (5.6) 415	416.8 (6.6) 417	424.3 (6.6) 423	
5-6	421.7 (6.8) 422	416.8 (4.7) 417	416.5 (6.2) 417	418.5 (6.5) 419	421.3 (6.9) 424	
6-7	423.3 (7.5) 424	418.1 (5.5) 419	418.5 (6.6) 419	418.4 (7.2) 420	427.1 (7.1) 426	
7-8	424.3 (8.6) 425	419.2 (6.6) 419	418.2 (6.7) 419	419.1 (7.6) 420	427.4 (6.5) 423	
8-9	424.3 (9.3) 427	421.7 (7.8) 421	418.3 (6.7) 422	420.8 (7.1) 423	426.7 (6.2) 423	421.4 (6.9)
9-10	428.0 (9.6) 428	424.2 (7.8) 424	420.9 (7.2) 421	423.7 (8.0) 424		424.3 (7.3)
9-11 (ACT)	429.5 (9.2) 428	425.5 (7.7) 424	422.1 (7.0) 421	425.3 (7.8) 424		425.7 (7.1)
10-11 (ACT)	429.8 (11.0) 431	425.5 (9.1) 427	421.1 (8.0) 423	424.1 (8.8) 426	426.3 (7.9) 427	425.6 (8.2)

Longitudinal test score correlations are provided in Table 7. For English and Mathematics, correlations for grades 3-4 through 7-8 are lower than those observed for grades 8-9, 9-10, and 9-11. Because the correlations for grades 8-9, 9-10, and 9-11 are based on the concordance-derived samples, we do not know if the correlation difference is due to a true difference in the longitudinal correlations across grade levels, or an artifact of using a concordance-derived sample. For English and Mathematics, the correlations generally increase with grade level. For Reading and Science, the correlations are very steady across grade levels, typically between 0.70 and 0.75. Across subject areas, the smallest correlations are observed for

Writing, ranging from 0.32 for grades 5-6 to 0.64 for grades 8-9. Given that the Writing tests require an essay response to a single writing prompt, the lower correlations are not surprising. Other possible reasons for the lower correlations for Writing include the changes across grade levels in the writing task and the use of rubric scoring. The Writing correlations are also subject to greater sampling error due to the smaller sample sizes. Correlations between grade 10 ACT Aspire scores and grade 11 ACT scores ranged from 0.47 for Writing to 0.84 for the Composite. Relative to Reading and Science, larger correlations were observed for English (0.79) and Mathematics (0.81).

Table 7

Longitudinal Correlations

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing	Composite
3-4	0.60	0.60	0.72	0.73	0.37	
4-5	0.66	0.58	0.72	0.72	0.47	
5-6	0.68	0.63	0.71	0.73	0.32	
6-7	0.67	0.64	0.70	0.75	0.47	
7-8	0.70	0.72	0.69	0.75	0.47	
8-9	0.81	0.78	0.74	0.73	0.64	0.89
9-10	0.81	0.81	0.73	0.74	--	0.89
9-11 (ACT)	0.81	0.81	0.73	0.72	--	0.88
10-11 (ACT)	0.79	0.81	0.66	0.65	0.47	0.84

Gain score means and standard deviations (Table 8) allow comparisons of gain scores across grade levels and subject areas. For all subject areas except Writing, all of the mean gain scores are positive, suggesting that students typically increase their Aspire-tested knowledge and skills after one year of schooling. The means and standard deviations of the gain scores vary considerably by grade level. The largest mean gain score was observed for grade 7-8 Science (4.6), while the smallest was observed for grade 6-7 Reading and grade 8-9 Reading (1.5).

Recall that for Writing, gain scores do not have a clear meaning and one should not necessarily expect positive mean gain scores because of the nature of the Writing scales. The mean gain scores for Writing ranged from 5.9 for grades 5-6 to -3.3 for grades 6-7. Because of the smaller sample sizes for Writing, the mean gain scores are subject to greater sampling error. Writing scores increased substantially from grade 5 to grade 6 (mean gain score of 5.9), and then declined from grade 6 to grade 7 (mean gain score of -3.3) and grade 7 to grade 8 (mean gain score of -2.8). One possible explanation for the increase in scores for grade 6 is that the grade 6 Writing test is a narrative writing exercise, unlike the grades 4 & 7 and 5 & 8 tests which are exercises in expository writing and persuasive argumentation, respectively. While raw scores of 4 indicate "proficient" across all grade levels, the data suggest that students performed better on the grade 6 narrative writing exercise, perhaps because of greater comfort and familiarity with the narrative mode.[4] At grade 6, the ACT Aspire Writing score range expands from 408-440 to 408-448, with a maximum raw score of 6 (instead of 5) for each domain score. It is likely that this scale increase is also partly responsible for the increase in scores from grade 5 to grade 6.

Table 8

Gain Score Means and Standard Deviations

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing	Composite
3-4	4.3 (5.3)	4.0 (3.7)	3.4 (4.1)	3.9 (4.6)	-0.2 (6.0)	
4-5	3.5 (5.5)	3.2 (4.4)	3.3 (4.4)	3.1 (4.8)	0.4 (6.5)	
5-6	2.7 (6.0)	3.4 (4.8)	3.5 (4.9)	2.6 (5.0)	5.9 (8.1)	
6-7	2.4 (6.8)	1.6 (5.4)	1.5 (5.0)	2.3 (5.3)	-3.3 (7.0)	
7-8	2.8 (6.6)	2.9 (5.4)	4.3 (5.3)	4.6 (5.4)	-2.8 (6.2)	
8-9	2.3 (5.9)	1.7 (5.1)	1.5 (5.1)	1.7 (5.6)	-1.0 (5.4)	1.8 (3.4)
9-10	2.5 (5.9)	2.7 (5.4)	2.7 (5.3)	2.0 (5.9)		2.5 (3.4)

[4] Chrys Dougherty suggested that narrative writing requires storytelling, which students may find inherently easier because the sequence of events in a story help students organize the information (personal communication, October 8, 2014).

The proportion of examinees with positive one-year gain scores is provided in Table 9. It is not uncommon for student scores to decline after one year. For all subjects except Writing, among all gain scores observed for grade level pairs 3-4 through 7-8, 21.8% were negative and 71.4% were positive. Similar to the gain score means, there is considerable variation across grade levels and subject areas. The highest proportion was observed for grade 3-4 Mathematics (0.83) and the lowest proportion was observed for grade 8-9 Reading and Science (0.54). It should be expected that grade level pairs and subject areas with larger mean gain scores would have a higher proportion of positive gain scores, and that the resulting predicted path estimates would include a greater share of positive gain scores.

For Writing, the proportion of examinees with positive one-year gain scores varied from 0.27 for grade level pairs 6-7 and 7-8 to 0.71 for grade level pair 5-6. Again, because of the nature of the Writing scale, one should not necessarily expect a large proportion of positive gain scores.

Table 9

Proportion with positive one-year gain scores

Grade Level Pair	Subject Area					
	English	Mathematics	Reading	Science	Writing	Composite
3-4	0.78	0.83	0.76	0.77	0.42	
4-5	0.71	0.72	0.74	0.70	0.45	
5-6	0.64	0.73	0.73	0.66	0.71	
6-7	0.63	0.57	0.59	0.64	0.27	
7-8	0.64	0.67	0.77	0.78	0.27	
8-9	0.58	0.55	0.54	0.54	0.33	0.66
9-10	0.61	0.62	0.66	0.57		0.74

Selected Predicted Path Graphical Results

Next, we examine line graphs with the predicted path results. With 43 sets of predicted paths (8 grade levels by 5 subject areas, plus Composite paths for grades 8-10), we only present selected results graphically. Figure 3 shows the one-year predicted paths for grade 3 ACT Aspire Science scores. The grade 3 scores are presented on the horizontal axis and predicted grade 4 scores are presented on the vertical axis. The blue line represents the lower bound of the predicted path and the red line represents the upper bound. The dashed black line represents the predicted grade 4 score that is used in the calculation of predicted mean scores for aggregate reports. Note that the predicted grade 4 scores are rounded to the nearest tenth, while the lower and upper bounds are rounded to the nearest integer. Therefore, the dashed black line appears smoother. The solid black line, provided for reference, represents the case where grade 3 score is equal to grade 4 score (gain score=0). All lines are monotonically increasing. The width of the one-year predicted path ranges from 5 (e.g., 427-431 for a grade 3 score of 428) to 8 (e.g., 409-416 for a grade 3 score of 407). The width of the predicted path decreases with higher grade 3 scores.

For grade 3 Science scores of 417 and lower, the paths predict *strictly positive* gain scores. This means that all values within the predicted path imply a positive gain score. For grade 3 Science scores of 424 and higher, the predicted paths include positive and negative gain scores. There are no grade 3 Science scores for which the paths predict *strictly negative* gain scores. Later, we examine the relative frequency with which the paths predict strictly positive and strictly negative gain scores. In Figure 3, the intersection of the grade 3 and grade 4 Benchmarks is marked with the green asterisk. The one-year predicted path for the grade 3 Benchmark includes the grade 4 Benchmark.

Figure 3

One-Year Predicted Paths for Grade 3 Science Scores

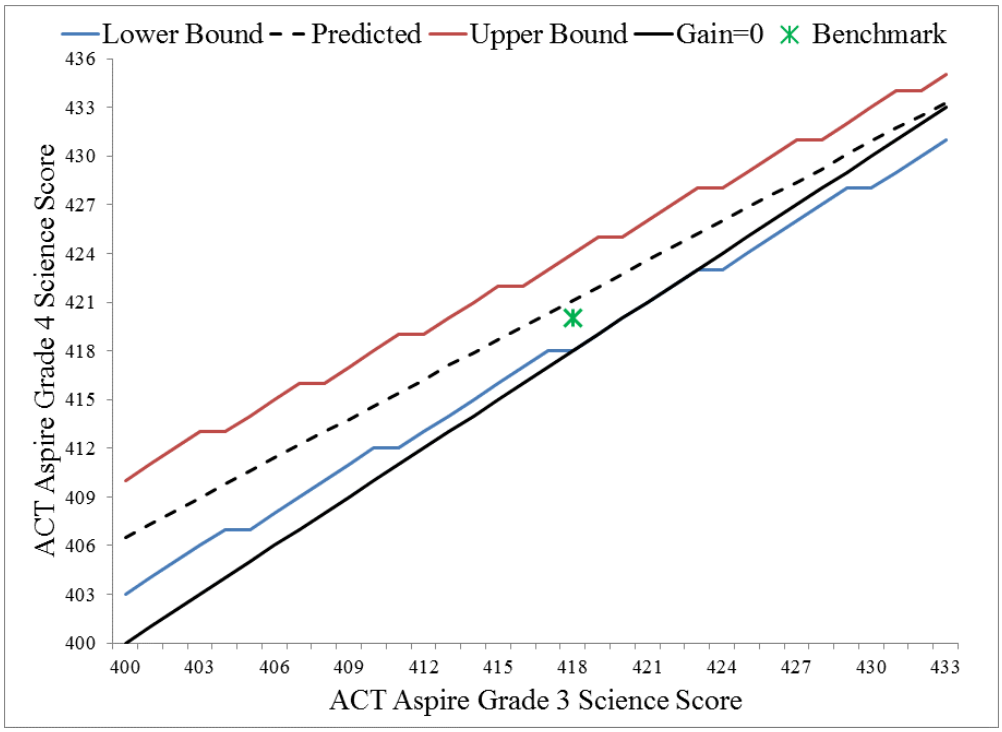


Figure 4 shows the two-year predicted paths for grade 3 ACT Aspire Science scores. By definition, the two-year predicted score ranges are twice as wide as the one-year ranges. Figure 4 also shows that a student who meets the grade 3 ACT Readiness Benchmark for Science (418) is predicted to score between 418 and 430 on the grade 5 ACT Aspire Science assessment. This range includes the grade 5 ACT Readiness Benchmark for Science (422).

Figure 4

Two-Year Predicted Paths for Grade 3 Science Scores

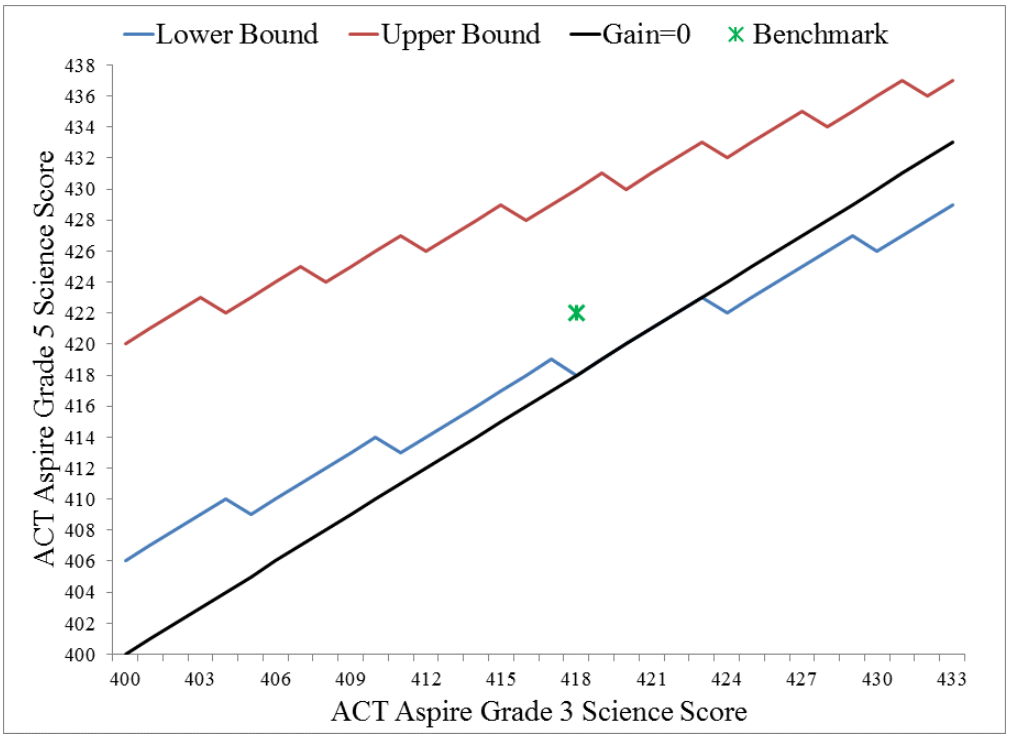


Figure 5 shows the one-year predicted paths for grade 8 Reading scores. In this case, the predicted paths were estimated using the concordance-derived longitudinal sample. The width of the one-year predicted path ranges from 6 to 8, with the width generally increasing with grade 8 Reading score. The sample standard deviation of grade 9 Reading scores was 7.2 (Table 6), so the width of the predicted path ranges from 0.83 to 1.11 standard deviations of grade 9 Reading scores. For grade 8 Reading scores of 407 and lower, the paths predict *strictly positive* gain scores. For grade 8 Reading scores of 413 and higher, the paths include negative gain scores. None of the grade 8 Reading paths predict *strictly negative* gain scores.

Figure 5

One-Year Predicted Paths for Grade 8 Reading Scores

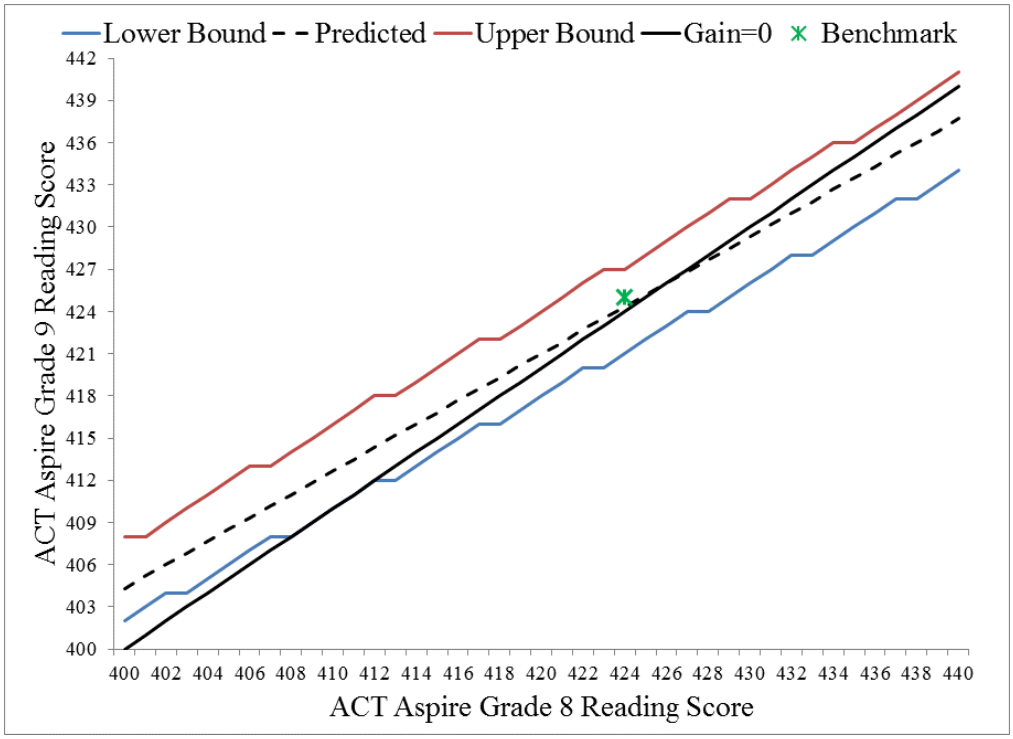


Figure 6 shows the two-year predicted paths for grade 8 Reading scores. Again, by definition, the two-year paths are twice as wide as the one-year paths. Students who score between 420 and 440 on the grade 8 Reading test will have a two-year predicted path that includes the grade 10 ACT Readiness Benchmark for Reading (428).

Figure 6

Two-Year Predicted Paths for Grade 8 Reading Scores

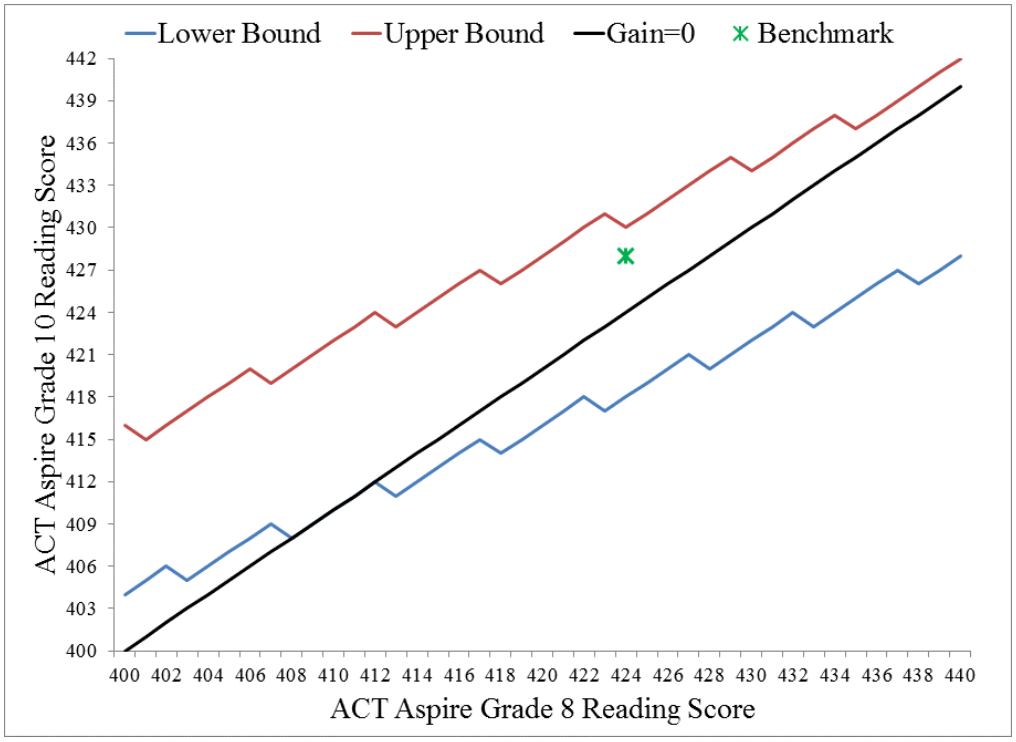
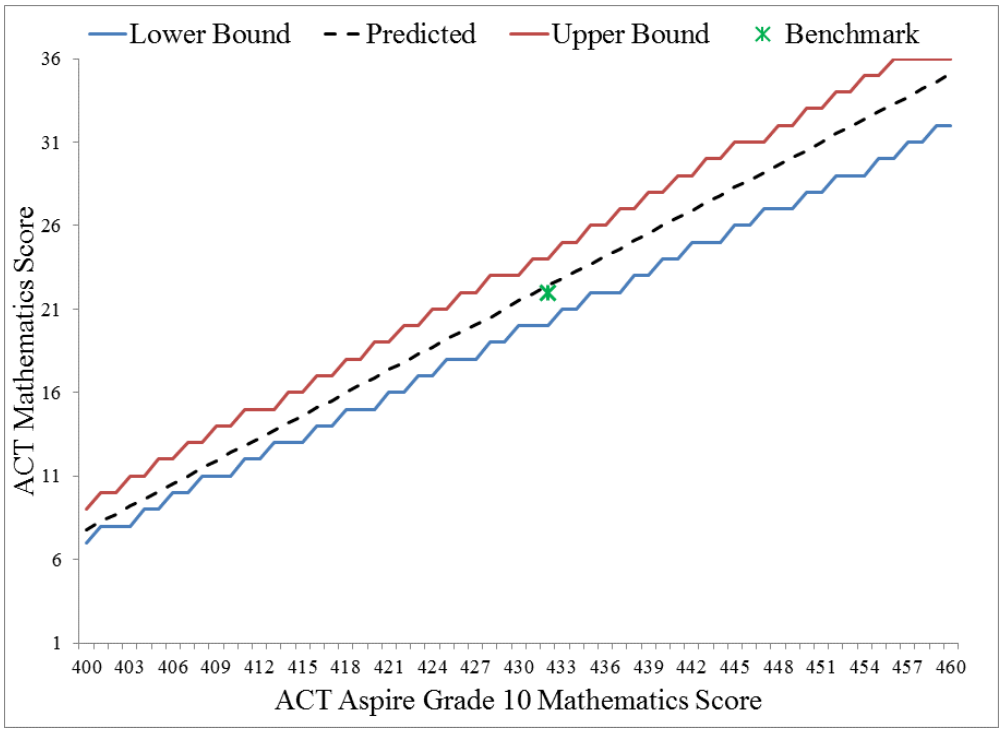


Figure 7 shows the predicted grade 11 ACT Mathematics score ranges for grade 10 Mathematics scores. The predicted paths were estimated using the longitudinal sample of students tested with ACT Aspire in spring 2013 and the ACT in spring 2014. The width of the predicted path increases as grade 10 Mathematics score increases. A student who meets the grade 10 ACT Readiness Benchmark for Mathematics (432) has a predicted ACT Mathematics score range of 20-24 with a predicted mean value of 22.4. As expected, the predicted score range for the grade 10 Benchmark includes the ACT College Readiness Benchmark for Mathematics (22).

Figure 7

Predicted ACT Mathematics Score Ranges for Grade 10 Mathematics Scores



Coverage Properties of Predicted Paths

Because the one-year predicted paths are estimated using the 25th and 75th percentiles of the conditional test score distributions, one would generally expect that 50% of students will score within the one-year predicted paths. However, using an inclusive definition of path coverage (that is, students who have a score equal to the 25th or 75th percentile are counted as within the interval), the expected percentage is greater than 50%. The one-year predicted paths should also have approximately symmetric coverage – that is, the percentages of students scoring above and below the predicted path should be similar.

Table 10 reports the percentage of students scoring below, within, and above the one-year predicted paths. These percentages are based on the ACT Aspire-tested longitudinal samples.

For grade level pairs 3-4, 4-5, 5-6, 6-7, and 7-8, we see that the percentage of students scoring within the one-year predicted path ranges from 54% (grade 6-7 English) to 60% (grade 6-7 Writing). For these grade level pairs, the predicted paths have mostly symmetric coverage. The largest level of asymmetry is observed for grade 6-7 Writing, where 22% of students scored below the predictive path and 18% scored above.

For grade 10-11 (ACT Aspire to the ACT), coverage rates range from 56% (for English) to 73% for Writing. The coverage rate for Writing is high because the ACT Writing test has only 11 possible score points, which leads to the 25^{th} and 75^{th} percentiles of the conditional score distribution encompassing a larger share of scores because of ties. The largest level of asymmetry is observed for Writing, where 16% of students scored below the predictive path and 11% scored above.

For grade level pairs 3-4, 4-5, 5-6, 6-7, 7-8, and 10-11, the same longitudinal samples used to estimate the predicted paths were used to evaluate the predicted paths. Thus, the level of coverage symmetry is likely higher than it would be if the estimation and evaluation samples were independent.

For grade level pairs 8-9 and 9-10, the concordance-derived longitudinal sample was used to estimate the predicted paths and the ACT Aspire-tested longitudinal sample was used to evaluate the predicted paths. The evaluation samples, while small, are independent of the estimation samples. The one-year predicted path coverage rates ranged from 43% (grade 9-10 Mathematics) to 87% (grade 9-10 Composite). There was less coverage symmetry than what was observed for the other grade level pairs. For example, for grade 8-9 Science, 64 students (41%) scored above their predicted path and 17 students (11%) scored below their predicted path. The large variability in coverage symmetry is expected from the small evaluation sample

sizes used for grades 8-9 and 9-10. In particular, for grades 9-10, the sample sizes ranged from only 15 (for Composite) to 26 (for English). Coverage asymmetry could also result from using the ACT Explore/ACT Plan to Aspire concordance tables.

Table 10

Number and percentage scoring below, within, and above one-year predicted path

Grade Level Pair	Score Location Relative to Path	Subject Area					
		English	Mathematics	Reading	Science	Writing	Composite
3-4	Below	797 (21%)	1,848 (21%)	1,877 (21%)	744 (22%)	106 (23%)	
	Within	2,162 (57%)	5,234 (59%)	5,124 (57%)	1,923 (57%)	263 (56%)	
	Above	825 (22%)	1,761 (20%)	1,942 (22%)	736 (22%)	100 (21%)	
4-5	Below	854 (22%)	1,825 (21%)	1,886 (22%)	679 (22%)	115 (24%)	
	Within	2,211 (57%)	4,987 (57%)	4,915 (57%)	1,734 (56%)	268 (55%)	
	Above	847 (22%)	1,912 (22%)	1,893 (22%)	657 (21%)	104 (21%)	
5-6	Below	779 (22%)	1,827 (23%)	1,744 (22%)	745 (22%)	90 (21%)	
	Within	1,976 (55%)	4,438 (56%)	4,562 (57%)	1,963 (57%)	245 (57%)	
	Above	808 (23%)	1,717 (22%)	1,702 (21%)	733 (21%)	96 (22%)	
6-7	Below	527 (23%)	1,286 (22%)	1,320 (22%)	369 (22%)	82 (22%)	
	Within	1,261 (54%)	3,293 (56%)	3,297 (56%)	948 (57%)	229 (60%)	
	Above	541 (23%)	1,297 (22%)	1,263 (21%)	360 (21%)	70 (18%)	
7-8	Below	428 (22%)	1,009 (22%)	1,023 (22%)	473 (22%)	55 (21%)	
	Within	1,070 (56%)	2,537 (55%)	2,573 (56%)	1,209 (57%)	154 (59%)	
	Above	426 (22%)	1,037 (23%)	983 (21%)	450 (21%)	53 (20%)	
8-9	Below	69 (29%)	57 (22%)	37 (31%)	17 (11%)	28 (24%)	6 (18%)
	Within	119 (49%)	142 (54%)	54 (46%)	75 (48%)	67 (58%)	19 (56%)
	Above	53 (22%)	62 (24%)	27 (23%)	64 (41%)	21 (18%)	9 (26%)
9-10	Below	1 (4%)	7 (33%)	4 (20%)	3 (14%)		1 (7%)
	Within	18 (69%)	9 (43%)	14 (70%)	13 (62%)		13 (87%)
	Above	7 (27%)	5 (24%)	2 (10%)	5 (24%)		1 (7%)
10-11 (ACT)	Below	913 (23%)	699 (19%)	747 (21%)	742 (21%)	144 (16%)	545 (19%)
	Within	2,225 (56%)	2,343 (62%)	2,014 (57%)	2,043 (58%)	675 (73%)	1,739 (61%)
	Above	854 (21%)	732 (19%)	779 (22%)	740 (21%)	103 (11%)	567 (20%)

Next, we evaluate the coverage of predicted paths for EPAS data (Table 11). While not directly applicable to ACT Aspire, these findings will allow us to examine the coverage of the predicted paths using separate estimation and evaluation data sets. The EPAS data also let us examine the coverage of the two-year predicted paths. The percentage of scores falling within

the one-year predicted path ranged from 56% (grade 10-11 Reading) to 68% (grade 9-10 Composite). The level of coverage symmetry varied across grade levels and subject areas. The largest levels of asymmetry were observed for grade 10-11 English (25% below, 16% above) and grade 8-9 Mathematics (22% below, 14% above). In these cases, it is likely that the 2014 cohort (the evaluation sample) demonstrated less growth than the 2013 cohort (the estimation sample), which would lead to a greater share of students scoring below their predicted path.

For the two-year predicted paths, the coverage rates were typically much higher than what was observed for the one-year predicted paths. The exception to this was grade 9-11 English, where the coverage rate was only 58%. The other two-year coverage rates ranged from 71% (grade 9-11 Reading) to 88% (grade 9-11 Mathematics). The largest degree of coverage asymmetry was observed for grade 9-11 English, with 12% scoring below the two-year predicted path and 30% scoring above. Generally, students are more likely to score above, rather than below, the two-year predicted path.

Table 11

Number and percent scoring within, above, and below one year and two-year predicted paths using EPAS data

Grade Level Pair	Score Location Relative to Path	Subject Area				
		English	Mathematics	Reading	Science	Composite
8-9	Below	10,127 (19%)	11,586 (22%)	10,630 (20%)	10,799 (20%)	9,885 (18%)
	Within	34,312 (64%)	34,191 (64%)	33,212 (62%)	34,312 (64%)	35,587 (66%)
	Above	9,098 (17%)	7,760 (14%)	9,695 (18%)	8,426 (16%)	8,065 (15%)
9-10	Below	30,074 (17%)	31,359 (18%)	37,075 (22%)	26,606 (15%)	23,743 (14%)
	Within	104,377 (61%)	115,077 (67%)	99,114 (58%)	110,986 (64%)	117,287 (68%)
	Above	37,888 (22%)	25,903 (15%)	36,150 (21%)	34,747 (20%)	31,309 (18%)
10-11	Below	36,591 (25%)	27,055 (18%)	33,683 (23%)	35,192 (24%)	31,604 (21%)
	Within	88,059 (59%)	89,711 (60%)	82,357 (56%)	88,655 (60%)	92,368 (62%)
	Above	23,706 (16%)	31,590 (21%)	32,316 (22%)	24,509 (17%)	24,384 (16%)
8-10 (2-year)	Below	29,231 (9%)	32,176 (10%)	25,653 (8%)	34,540 (11%)	27,935 (9%)
	Within	274,305 (85%)	248,898 (77%)	245,789 (76%)	253,167 (78%)	259,021 (80%)
	Above	20,205 (6%)	42,667 (13%)	52,299 (16%)	36,034 (11%)	36,785 (11%)
9-11 (2-year)	Below	6,057 (12%)	2,486 (5%)	5,171 (10%)	5,120 (10%)	3,077 (6%)
	Within	29,236 (58%)	44,775 (88%)	35,736 (71%)	36,596 (72%)	39,352 (78%)
	Above	15,363 (30%)	3,395 (7%)	9,749 (19%)	8,940 (18%)	8,227 (16%)

Next, we examine how often the predicted paths include positive (or negative) gain scores. In Table 12, we present the percentage of Aspire-tested students of spring 2013 whose predicted path would be strictly positive (include only positive gain scores), strictly negative (include strictly negative gain scores), or other.[5] (For this analysis, we do not need to distinguish the one-year paths from the two-year paths because the two-year path is extrapolated from the one-year path, maintaining the same positive or negative trend). There is considerable variation in the percentages across grade levels and subject areas. Generally, the percentage of predicted paths that are strictly positive is larger for cases where positive gain scores are more likely. For example, the highest percentages for strictly positive correspond to grade 3 Mathematics (82%), grade 3 Science (74%), and grade 7 Science (71%), which also had among the highest

[5] Referring back to Figures 3, 5, and 7, strictly positive (negative) cases occur when the prediction interval lies entirely above (below) the gain=0 line.

proportions with positive gain scores (Table 9). Conversely, lowest percentages for strictly positive were observed for grade 6 English (1%), grade 6 Mathematics (1%), grade 8 English (1%), and grade 8 Science (3%), which also had the lowest proportions with positive gain scores. Low percentages of strictly positive predicted paths for grade 8 are due to the modest gains observed between grades 8 and 9.

In some rare cases, strictly negative predicted paths are reported. This was most common for Writing, which is expected because of its scale construction and the differences across grade level pairs in mean gain scores. Among the other subject areas, strictly negative paths were most common for grade 6 Reading, with 2% of the paths predicting strictly negative gain scores. This is explained by the relatively small gain scores (mean=1.5, sd=5.0) observed for grade 6-7 Reading (Table 8).

Table 12

Number and percentage of students whose predicted path includes strictly positive or strictly negative gain

Grade Level	Gain Prediction	Subject Area					
		English	Mathematics	Reading	Science	Writing	Composite
3	Positive	11,155 (67%)	14,631 (82%)	11,422 (64%)	12,218 (74%)	1,403 (21%)	
	Other	5,511 (33%)	3,160 (18%)	6,358 (36%)	4,199 (26%)	4,151 (62%)	
	Negative	0 (0%)	0 (0%)	0 (0%)	0 (0%)	1,155 (17%)	
4	Positive	7,067 (41%)	6,399 (36%)	9,180 (54%)	8,334 (51%)	1,949 (32%)	
	Other	9,979 (59%)	11,170 (64%)	7,950 (46%)	8,106 (49%)	3,244 (53%)	
	Negative	0 (0%)	0 (0%)	0 (0%)	10 (<0.5%)	971 (16%)	
5	Positive	1,964 (12%)	8,361 (50%)	7,698 (47%)	3,430 (22%)	2,571 (41%)	
	Other	14,219 (88%)	8,415 (50%)	8,721 (53%)	12,365 (78%)	3,640 (58%)	
	Negative	0 (0%)	0 (0%)	0 (0%)	0 (0%)	96 (2%)	
6	Positive	79 (1%)	159 (1%)	3,621 (25%)	1,107 (8%)	503 (8%)	
	Other	14,388 (99%)	14,802 (99%)	10,362 (73%)	12,960 (92%)	3,306 (51%)	
	Negative	0 (0%)	0 (0%)	241 (2%)	0 (0%)	2,713 (42%)	
7	Positive	2,726 (21%)	1,560 (12%)	8,196 (64%)	9,149 (71%)	805 (12%)	
	Other	10,191 (79%)	11,483 (88%)	4,638 (36%)	3,749 (29%)	3,961 (60%)	
	Negative	24 (<0.5%)	0 (0%)	0 (0%)	0 (0%)	1,834 (28%)	
8	Positive	78 (1%)	2,057 (19%)	430 (4%)	334 (3%)	1,615 (32%)	0 (0%)
	Other	10,579 (99%)	9,034 (81%)	10,352 (96%)	10,515 (97%)	3,173 (63%)	12,061 (100%)
	Negative	0 (0%)	5 (<0.5%)	0 (0%)	0 (0%)	264 (5%)	0 (0%)
9	Positive	2,103 (26%)	702 (9%)	2,644 (35%)	827 (11%)	1,084 (35%)	5,433 (56%)
	Other	6,125 (74%)	7,105 (91%)	4,936 (65%)	6,906 (89%)	1,814 (59%)	4,204 (44%)
	Negative	18 (<0.5%)	0 (0%)	18 (<0.5%)	0 (0%)	202 (7%)	0 (0%)

Summary

Interpretations and limitations of the predicted paths

The ACT Aspire predicted paths are designed to predict students' next-year score ranges with greater than 50% accuracy and with approximate coverage symmetry (e.g., equal percentages of scores over- and under-predicted). The predicted paths cover two years, with the predicted score range for the second year formed by extending the one-year path in a linear fashion. Based on the EPAS data presented in Table 11, it is expected that the two-year coverage rates will be considerably higher than the one-year rates, with under-prediction more likely than over-prediction.

Given the design of the predicted paths, they support the following uses:

- To determine if students are likely to meet ACT Readiness Benchmarks over the next two years
- To determine which students are unlikely to meet a future-year standard and thus are candidates for extra academic support
- To predict aggregate future achievement for a classroom, school, district, or state
- To predict student achievement on the ACT (for grades 9 and 10)

Students who score above their one-year predicted path will have demonstrated greater-than-expected growth; students who score below their one-year predicted path will have demonstrated less-than-expected growth. Generally, students who score above their one-year predicted path will have achieved a student growth percentile (Betebenner, 2008) greater than 75; students who score below their one-year predicted path will have achieved a student growth percentile less than 25.

The predicted paths for grades 3-7 and grade 10 were developed using students tested in spring 2013 and spring 2014. The predicted paths for grades 8 and 9 were developed using students tested 10-14 (or 22-26) months apart. Thus, interpretations of the one-year paths should assume a 12 month testing interval and interpretations of the two-year paths should assume a 24 month testing interval. For students who test in the fall, the predicted paths assume that they will test again in the fall the following year. For example, if a student tests in fall grade 10 and plans to take the ACT in spring grade 11, their predicted ACT score range is likely a slight underestimate because there is more time remaining for growth than is assumed by the prediction.

Limitations of the predicted paths include:

- Sample dependence. The predicted paths for grades 3-7 and grade 10 are developed using samples of students tested in spring 2013 and spring 2014, with a large percentage coming from one state (Alabama, the first state to administer ACT Aspire on a statewide basis). With larger samples of students, the estimation of the predicted paths will have less sampling error. It is also possible that predicted path estimates could shift upwards or downwards with the inclusion of more school districts and states in the sample. The predicted paths will be re-estimated as more data become available.
- Concordance dependence. The predicted paths for grades 8 and 9 were developed using large samples of students tested with ACT's EPAS system, with greater geographic diversity. However, this approach relies on the ACT Explore/ACT Plan to ACT Aspire concordance, which could introduce bias into the estimation of the predicted paths. As more data become available, the predicted paths for grades 8 and 9 will be estimated using ACT Aspire data without the use of the concordance.
- The two-year predicted score ranges, defined as a linear extension of the one-year ranges, will likely have asymmetric coverage.

Future studies will monitor the predictive performance of the predicted paths and seek to identify steps that can be taken to improve the accuracy and interpretive value of the predicted paths.

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