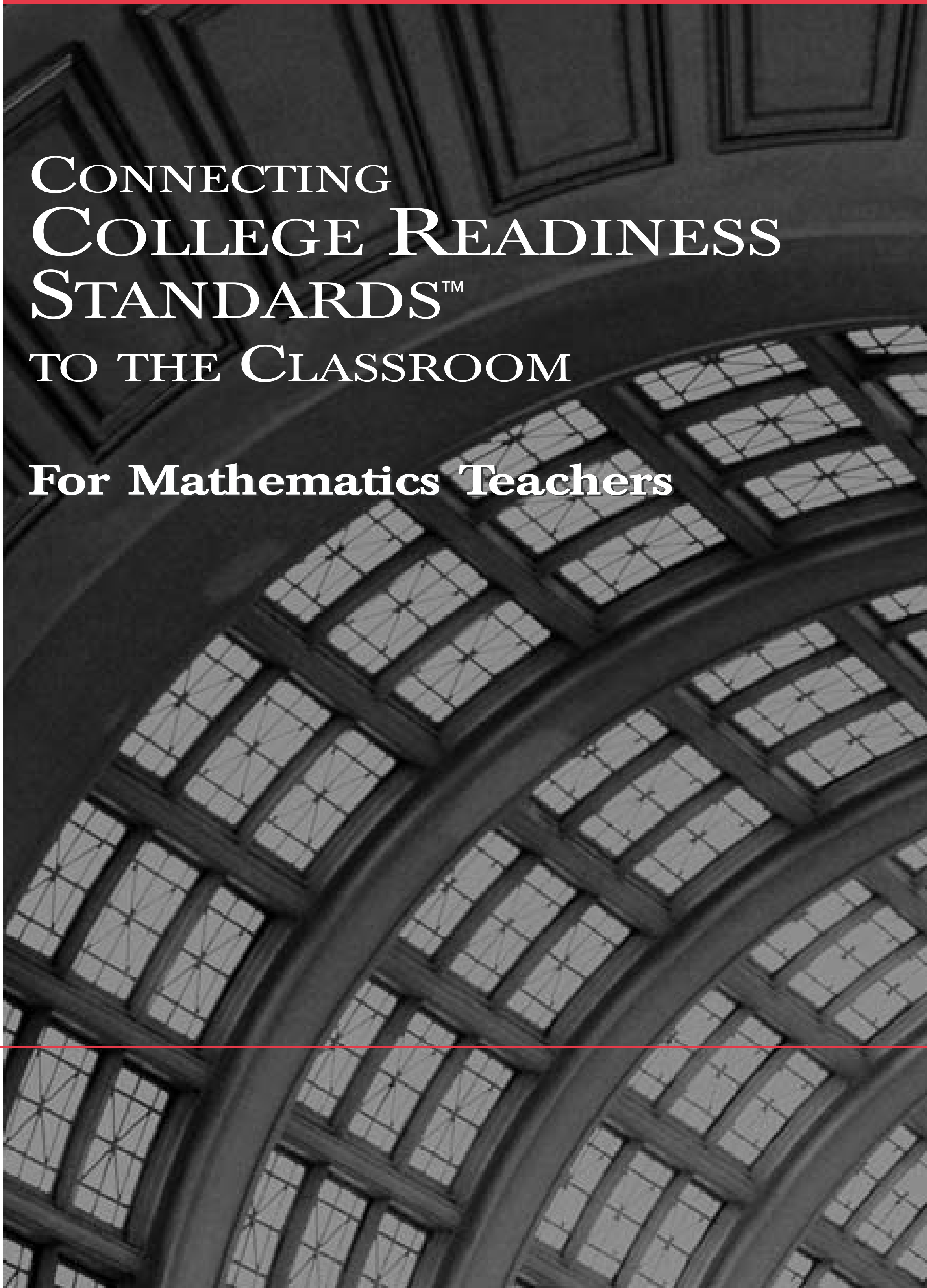
PLAN®



Connecting College Readiness Standards™ to the Classroom

For Mathematics Teachers

ACT®

ACT endorses the *Code of Fair Testing Practices in Education* and the *Code of Professional Responsibilities in Educational Measurement*, guides to the conduct of those involved in educational testing. ACT is committed to ensuring that each of its testing programs upholds the guidelines in each *Code*.

A copy of each *Code* may be obtained free of charge from ACT Customer Services (68), P.O. Box 1008, Iowa City, IA 52243-1008, 319/337-1429.

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INTRODUCTION

ACT has developed this guide to help classroom teachers, curriculum coordinators, and counselors interpret the College Readiness Standards Report data for PLAN® Mathematics. The guide includes:

- A description of the **College Readiness Standards**™ and **Benchmarks** for PLAN
- A description of the **PLAN Mathematics Test**
- A set of **sample test questions**
- A description of the **Assessment-Instruction Link**
- A set of classroom **instructional activities**

The College Readiness Standards for PLAN are statements that describe what students who score in the five score ranges 13–15, 16–19, 20–23, 24–27, and 28–32 are *likely* to know and to be able to do. The statements are generalizations based on the performance of many students scoring in these five score ranges. College Readiness Standards have not been developed for students whose scores fall in the 1–12 range because these students, as a group, do not demonstrate skills similar to each other consistently enough to permit useful generalizations.

The College Readiness Standards for PLAN are accompanied by ideas for progress that help teachers identify ways of enhancing student learning based on the scores students receive.

The College Readiness Standards Report for PLAN provides the percentage of your students in each College Readiness Standards score range in each of the four content areas the PLAN test measures—English, Mathematics, Reading, and Science. The report provides data that compare the performance of your students (Local) with all students in a nationally representative comparison group (norm group).

PLAN is a curriculum-based assessment program developed by ACT to help tenth graders plan their academic careers and prepare for entry into college or the world of work. As part of ACT's Educational Planning and Assessment System (EPAS®), PLAN is complemented by EXPLORE®, ACT's eighth- and ninth-grade program, and by the ACT®, for eleventh and twelfth graders. We hope this guide helps you assist your students as they plan and pursue their future studies.

> **"The role of standardized testing is to let parents, students, and institutions know what students are ready to learn next."**
>
> — Ralph Tyler, October 1991
> Chairman Emeritus of
> ACT's Board of Trustees

The College Readiness Standards Report for PLAN Mathematics

The College Readiness Standards Report data for PLAN Mathematics allow you to compare the performance of students in your school with the performance of students nationwide. The report provides summary information you can use to map the development of your students' knowledge and skills in mathematics. Used along with your own classroom observations and with other resources, the test results can help you to analyze your students' progress in mathematics and to identify areas of strength and areas that need more attention to ensure your students are on track to be college ready by the time they graduate from high school. You can then use the Standards as one source of information in the instructional planning process.

A sample report appears on the next page. An explanation of its features is provided below.

College Readiness Standards Ranges

Down the sides of the report, in shaded boxes, are the six score ranges reported for the College Readiness Standards for PLAN. To determine the number of score ranges and the width of each score range, ACT staff reviewed normative data, college admission criteria, and information obtained through ACT's Course Placement Service. For a more detailed explanation of the way the score ranges were determined, see page 5. For a table listing the College Readiness Standards by score range for Mathematics, see page 8. For a discussion of College Readiness Benchmark Scores, see page 38.

Local and National Student Results

In the center of the report, the percent of students who scored in a particular score range at an individual school (Local) is compared with the percent of all tenth-grade students in the norm group (National) who scored in the same range. The percent of students for the norm group is based on the most current set of nationally representative norms.

The College Readiness Standards

The College Readiness Standards were developed by identifying the knowledge and skills students need in order to respond successfully to questions on the PLAN Mathematics Test. The Standards are cumulative, which means that if students score, for example, in the 20–23 score range, they are likely to be able to demonstrate most or all of the knowledge and skills in the 13–15, 16–19, and 20–23 score ranges. Students may be able to demonstrate some of the skills in the next score range, 24–27, but not consistently enough as a group to reach that score range. A description of the way the College Readiness Standards were developed can be found on pages 5–6. A table listing the College Readiness Standards for Mathematics can be found on page 8.

2009-2010 PLAN Profile Summary Report	Page: 4
School Report -	Code: 999999
Custom Report	HIGH SCHOOL Name
Custom Description	CITY, ST
National Norm Group: Fall 10th	Total Students In Report: 113

TABLE 1c: Are our students *On Track* to be college ready when they graduate from high school?

College Readiness Standards Report
(Percent of students in College Readiness Standards score ranges)

CRS Range	English (Benchmark =15)	Mathematics (Benchmark =19)	Reading (Benchmark = 17)	Science (Benchmark = 21)	CRS Range
1-12	15 17	5 7	12 15	3 4	1-12
13-15	18 21	23 25	25 27	15 17	13-15
16-19	42 36	44 40	28 25	43 44	16-19
20-23	17 19	18 15	20 21	27 27	20-23
24-27	6 6	8 7	10 9	8 6	24-27
28-32	2 1	2 5	5 2	3 2	28-32
% At or Above Benchmark	72 69	36 35	56 51	25 25	% At or Above Benchmark

= Local = National

Notes: *Connecting College Readiness Standards to the Classroom* interpretive guides to use with this report can be found at www.act.org/standard/guides/plan. The College Readiness Standards can be found starting on page 8 of each content guide.

Local report group percentages can be compared with national reference group percentages, which are based on of all 10th-grade students in the norm group. All percentages have been rounded to the nearest whole number.

The score ranges given in this report are linked to the College Readiness Standards, which describe what students who score in various score ranges are likely to know and to be able to do, and reflect the progression and complexity of skills in each test of the PLAN program. College Readiness Benchmark Scores have been developed for PLAN to indicate a student's probable readiness for entry-level college coursework by the time the student graduates from high school. The data from this report, along with the College Readiness Standards and Benchmarks and information from other sources, can be used to inform local instructional priorities.

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C#: 11716
PN#: 99999999

ACT

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Description of the College Readiness Standards

What Are the College Readiness Standards?

The College Readiness Standards communicate educational expectations. Each Standard describes what students who score in the designated range are *likely* to be able to do with what they know. Students can typically demonstrate the skills and knowledge within the score ranges preceding the range in which they scored, so the College Readiness Standards are cumulative.

In helping students make the transition from high school to postsecondary education or to the world of work, teachers, counselors, and parents can use the College Readiness Standards for PLAN to interpret students' scores and to understand which skills students need to develop to be better prepared for the future.

How Were the Score Ranges Determined?

To determine the number of score ranges and the width of each score range for PLAN, ACT staff reviewed PLAN normative data and considered the relationship among EXPLORE, PLAN, and the ACT.

In reviewing the PLAN normative data, ACT staff analyzed the distribution of student scores across the score scale. Because PLAN and the ACT have a common score scale, ACT can provide PLAN examinees with an estimated ACT Composite score. When the score ranges were being determined, therefore, both the PLAN score scale, 1–32, and the ACT score scale, 1–36, were reviewed side by side. And because many students take PLAN to determine how well they might perform on the ACT, the course-placement research that ACT has conducted over the last forty years was also reviewed. ACT's Course Placement Service provides colleges and universities with cutoff scores that are used to place students into appropriate entry-level courses in college; and these cutoff scores were used to help define the score ranges.

After analyzing all the data and reviewing different possible score ranges, ACT staff concluded that using the six score ranges 1–12, 13–15, 16–19, 20–23, 24–27, and 28–32 would best distinguish students' levels of achievement so as to assist teachers, administrators, and others in relating PLAN test scores to students' attainment of specific skills and understandings.

How Were the College Readiness Standards Developed?

After reviewing normative data, college admission criteria, and information obtained through ACT's Course Placement Service, content experts wrote the College Readiness Standards based on their analysis of the skills and knowledge students need in order to successfully respond to the test questions in each score range. Experts analyzed numerous test questions that had been answered correctly by 80%

> **"The examination should describe the student in meaningful terms–meaningful to the student, the parent, and the elementary and high school teacher–meaningful in the sense that the profile scores correspond to recognizable school activities, and directly suggest appropriate distributions of emphasis in learning and teaching."**
>
> — E. F. Lindquist, February 1958
> Cofounder of ACT

or more of the examinees within each score range. The 80% criterion was chosen because it offers those who use the College Readiness Standards a high degree of confidence that students scoring in a given score range will most *likely* be able to demonstrate the skills and knowledge described in that range.

As a content validity check, ACT invited nationally recognized scholars from high school and university Mathematics and Education departments to review the College Readiness Standards for the PLAN Mathematics Test. These teachers and researchers provided ACT with independent, authoritative reviews of the ways the College Readiness Standards reflect the skills and knowledge students need to successfully respond to the questions on the PLAN Mathematics Test.

Because PLAN is curriculum based, ACT and independent consultants conduct a review every three to four years to ensure that the knowledge and skills described in the Standards and outlined in the test specifications continue to reflect those being taught in classrooms nationwide.

How Should the College Readiness Standards Be Interpreted and Used?

The College Readiness Standards reflect the progression and complexity of the skills measured in PLAN. Because no PLAN test form measures all of the skills and knowledge included in the College Readiness Standards, the Standards must be interpreted as skills and knowledge that *most* students who score in a particular score range are *likely* to be able to demonstrate. Since there were relatively few test questions that were answered correctly by 80% or more of the students who scored in the lower score ranges, the Standards in these ranges should be interpreted cautiously. The skills and understandings of students who score in the 1–12 score range may still be evolving. For these students the skills and understandings in the higher score ranges could become their target achievement outcomes.

It is important to recognize that PLAN does not measure everything students have learned nor does any test measure everything necessary for students to know to be successful in college or in the world of work. The PLAN Mathematics Test includes questions from a large domain of skills and from areas of knowledge that have been judged important for success in college and beyond. Thus, the College Readiness Standards should be interpreted in a responsible way that will help students understand what they need to know and do if they are going to make a successful transition to college, vocational school, or the world of work. As students choose courses they plan to take in high school, they can use the Standards to identify the skills and knowledge they need to develop to be better prepared for their future. Teachers and curriculum coordinators can use the Standards to learn more about their students' academic strengths and weaknesses and can then modify their instruction and guide students accordingly.

How Are the College Readiness Standards Organized?

As content experts reviewed the test questions connected to each score range, distinct yet overlapping areas of knowledge and skill were identified. For example, there are many types of questions in which students are asked to solve arithmetic problems. Therefore, *Basic Operations & Applications* is one area, or strand, within the College Readiness Standards for PLAN Mathematics. The other strands are *Probability, Statistics, & Data Analysis; Numbers: Concepts & Properties; Expressions, Equations, & Inequalities; Graphical Representations; Properties of Plane Figures;* and *Measurement.*

The strands provide an organizational framework for the College Readiness Standards statements. As you review the Standards, you will note a progression in complexity within each strand. For example, in the 13–15 range for the Basic Operations & Applications strand, students are able to "solve problems in one or two steps using whole numbers," while in the 28–32 range, students demonstrate that they are able to "solve word problems containing several rates, proportions, or percentages."

The Standards are complemented by brief descriptions of learning experiences from which high school students might benefit. Based on the College Readiness Standards, these ideas for progress are designed to provide classroom teachers with help for lesson plan development. These ideas, which are given in Table 1, demonstrate one way that information learned from standardized test results can be used to inform classroom instruction.

Because students learn over time and in various contexts, it is important to use a variety of instructional methods and materials to meet students' diverse needs and to help strengthen and build upon their knowledge and skills. The ideas for progress offer teachers a variety of suggestions to foster learning experiences from which students would likely benefit as they move from one level of learning to the next.

Because learning is a complex and individual process, it is especially important to use multiple sources of information—classroom observations and teacher-developed assessment tools, as well as standardized tests—to accurately reflect what each student knows and can do. The Standards and the ideas for progress, used in conjunction with classroom-based and curricular resources, help teachers and administrators to guide the whole education of every student.

What Are the PLAN Mathematics Test College Readiness Standards?

Table 1 on pages 8–15 suggests links between what students are *likely* to be able to do (the College Readiness Standards) and what learning experiences students would likely benefit from.

The College Readiness Standards are organized both by score range (along the left-hand side) and by strand (across the top). The lack of a College Readiness Standards statement in a score range indicates that there was insufficient evidence with which to determine a descriptor.

The ideas for progress are also arranged by score range and by strand. Although many of the ideas cross more than one strand, a primary strand has been identified for each in order to facilitate their use in the classroom. For example, the statement in the 20–23 score range "represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)" brings together concepts from several strands, such as Expressions, Equations, & Inequalities, and Graphical Representations. However, this idea is primarily linked to the Graphical Representations strand.

As you review the table, you will note that ideas for progress have been provided for the 28–32 score range, the highest score range for PLAN. PLAN is designed to measure knowledge and skills achieved through the tenth grade. Ideas for progress for the 28–32 score range are shown to suggest educational experiences from which students may benefit before they take the ACT.

PLAN MATHEMATICS TEST

Table 1: The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive. The score range at the Benchmark level of achievement is highlighted.

Score range		Basic Operations & Applications	Probability, Statistics, & Data Analysis	Numbers: Concepts & Properties
1–12	**Standards**	■ Students who score in the 1–12 range are most likely beginning to develop the knowledge and skills assessed in the other score ranges.		
	ideas for progress	■ practice and apply estimation and computation using whole numbers and decimals ■ choose the appropriate method of computation to solve multistep problems (e.g., calculator, mental, or pencil and paper) ■ practice selecting appropriate units of measure (e.g., inches or feet, hours or minutes, centimeters or meters) and converting between units ■ model and connect physical, verbal, and symbolic representations of money	■ interpret data from a variety of displays and use it in computation (e.g., mean, median, mode, range) ■ organize, display, and analyze data in a variety of ways	
13–15	**Standards**	■ Perform one-operation computation with whole numbers and decimals ■ Solve problems in one or two steps using whole numbers ■ Perform common conversions (e.g., inches to feet or hours to minutes)	■ Calculate the average of a list of positive whole numbers ■ Perform a single computation using information from a table or chart	■ Recognize equivalent fractions and fractions in lowest terms
	ideas for progress	■ investigate and build understanding of the concept of percentage as a comparison of a part to a whole ■ use multiple operations to solve multistep arithmetic problems	■ solve real-world problems that involve measures of central tendency (e.g., mean, median, mode) ■ interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems ■ conduct simple probability experiments and represent results using different formats	■ recognize and apply place value, rounding, and elementary number theory concepts

Expressions, Equations, & Inequalities	Graphical Representations	Properties of Plane Figures	Measurement
■ model a variety of problem situations with expressions and/or equations ■ use the inverse relationships for the basic operations of addition and subtraction to determine unknown quantities	■ locate and describe points in terms of their position on the number line		■ identify line segments in geometric figures and estimate or calculate their measure
■ Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) ■ Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals	■ Identify the location of a point with a positive coordinate on the number line		■ Estimate or calculate the length of a line segment based on other lengths given on a geometric figure
■ use mathematical symbols and variables to express a relationship between quantities (e.g., the number of 59¢ candy bars that you can buy for $5 must satisfy $59n \leq 500$) ■ evaluate algebraic expressions and solve simple equations using integers	■ locate and describe objects in terms of their position on the number line and on a grid	■ describe, compare, and contrast plane and solid figures using their attributes	■ distinguish between area and perimeter, and find the area or perimeter when all relevant dimensions are given

PLAN MATHEMATICS TEST

Table 1 (continued): **The College Readiness Standards**				
		The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive. The score range at the Benchmark level of achievement is highlighted.		
		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
16–19	**Standards**	■ Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent ■ Solve some routine two-step arithmetic problems	■ Calculate the average of a list of numbers ■ Calculate the average, given the number of data values and the sum of the data values ■ Read tables and graphs ■ Perform computations on data from tables and graphs ■ Use the relationship between the probability of an event and the probability of its complement	■ Recognize one-digit factors of a number ■ Identify a digit's place value
	ideas for progress	■ solve routine arithmetic problems that involve rates, proportions, and percents ■ model and solve problems that contain verbal and symbolic representations of money ■ do multistep computations with rational numbers	■ interpret data and use appropriate measures of central tendency to find unknown values ■ find the probability of a simple event in a variety of settings ■ gather, organize, display, and analyze data in a variety of ways to use in problem solving ■ conduct simple probability experiments, use a variety of counting techniques (e.g., Venn diagrams, Fundamental Counting Principle, organized lists), and represent results from data using different formats	■ apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring ■ recognize, identify, and apply field axioms (e.g., commutative)
20–23	**Standards**	■ Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	■ Calculate the missing data value, given the average and all data values but one ■ Translate from one representation of data to another (e.g., a bar graph to a circle graph) ■ Determine the probability of a simple event ■ Exhibit knowledge of simple counting techniques	■ Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor
	ideas for progress	■ apply and use number properties to model and solve problems that involve reasoning with proportions ■ select and use appropriate units when solving problems that involve one or more units of measure	■ construct and analyze Venn diagrams to help determine simple probabilities	■ use the inverse relationships for the four basic operations, exponentiation, and root extractions to determine unknown quantities

Expressions, Equations, & Inequalities	Graphical Representations	Properties of Plane Figures	Measurement
■ Substitute whole numbers for unknown quantities to evaluate expressions ■ Solve one-step equations having integer or decimal answers ■ Combine like terms (e.g., $2x + 5x$)	■ Locate points on the number line and in the first quadrant	■ Exhibit some knowledge of the angles associated with parallel lines	■ Compute the perimeter of polygons when all side lengths are given ■ Compute the area of rectangles when whole number dimensions are given
■ create expressions that model mathematical situations using combinations of symbols and numbers ■ evaluate algebraic expressions and solve multistep first-degree equations	■ sketch and identify line segments, midpoints, intersections, and vertical and horizontal lines	■ describe angles and triangles using mathematical terminology and apply their properties	■ find area and perimeter of a variety of polygons by substituting given values into standard geometric formulas
■ Evaluate algebraic expressions by substituting integers for unknown quantities ■ Add and subtract simple algebraic expressions ■ Solve routine first-degree equations ■ Perform straightforward word-to-symbol translations ■ Multiply two binomials	■ Locate points in the coordinate plane ■ Comprehend the concept of length on the number line ■ Exhibit knowledge of slope	■ Find the measure of an angle using properties of parallel lines ■ Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)	■ Compute the area and perimeter of triangles and rectangles in simple problems ■ Use geometric formulas when all necessary information is given
■ identify, interpret, and generate symbolic representations that model the context of a problem ■ factor and perform the basic operations on polynomials ■ create and solve linear equations and inequalities that model real-world situations ■ solve literal equations for any variable	■ represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)	■ recognize what geometric properties and relationships for parallel lines to apply to find unknown angle measures ■ recognize when to apply geometric properties and relationships of triangles to find unknown angle measures	■ apply a variety of strategies to determine the circumference or perimeter and the area for circles, triangles, rectangles, and composite geometric figures

PLAN MATHEMATICS TEST

		Table 1 (continued): **The College Readiness Standards**		
		The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive. The score range at the Benchmark level of achievement is highlighted.		
		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
24–27	**Standards**	■ Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	■ Calculate the average, given the frequency counts of all the data values ■ Manipulate data from tables and graphs ■ Compute straightforward probabilities for common situations ■ Use Venn diagrams in counting	■ Find and use the least common multiple ■ Order fractions ■ Work with numerical factors ■ Work with scientific notation ■ Work with squares and square roots of numbers ■ Work problems involving positive integer exponents ■ Work with cubes and cube roots of numbers ■ Determine when an expression is undefined
	ideas for progress	■ model and solve real-world problems that involve a combination of rates, proportions, and/or percents	■ find the probability of simple events, disjoint events, compound events, and independent events in a variety of settings using a variety of counting techniques	■ apply and use elementary number concepts and number properties to model and solve nonroutine problems that involve new ideas

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*
■ Solve real-world problems using first-degree equations ■ Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) ■ Identify solutions to simple quadratic equations ■ Add, subtract, and multiply polynomials ■ Factor simple quadratics (e.g., the difference of squares and perfect square trinomials) ■ Solve first-degree inequalities that do not require reversing the inequality sign	■ Identify the graph of a linear inequality on the number line ■ Determine the slope of a line from points or equations ■ Match linear graphs with their equations ■ Find the midpoint of a line segment	■ Use several angle properties to find an unknown angle measure ■ Recognize Pythagorean triples ■ Use properties of isosceles triangles	■ Compute the area of triangles and rectangles when one or more additional simple steps are required ■ Compute the area and circumference of circles after identifying necessary information ■ Compute the perimeter of simple composite geometric figures with unknown side lengths
■ create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings ■ explore and use different methods to solve systems of equations ■ manipulate radical expressions (e.g., rationalize denominators)	■ graph linear equations and inequalities, determine slopes of lines, identify parallel and perpendicular lines, and find distances ■ identify characteristics of figures from a general equation	■ apply special right-triangle properties and the Pythagorean theorem to solve congruent and similar shape problems	■ apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures

PLAN MATHEMATICS TEST

Table 1 (continued): **The College Readiness Standards**				
		The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive. The score range at the Benchmark level of achievement is highlighted.		
		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
28–32	**Standards**	■ Solve word problems containing several rates, proportions, or percentages	■ Calculate or use a weighted average ■ Interpret and use information from figures, tables, and graphs ■ Apply counting techniques ■ Compute a probability when the event and/or sample space are not given or obvious	■ Apply number properties involving prime factorization ■ Apply number properties involving even/odd numbers and factors/multiples ■ Apply number properties involving positive/negative numbers ■ Apply rules of exponents
	ideas for progress	■ solve problems that require combining multiple concepts	■ design and conduct probability investigations (e.g., how the margin of error is determined) and then determine, analyze, and communicate the results	■ explain, solve, and/or draw conclusions for complex problems using relationships and elementary number concepts

Expressions, Equations, & Inequalities	Graphical Representations	Properties of Plane Figures	Measurement
■ Manipulate expressions and equations ■ Write expressions, equations, and inequalities for common algebra settings ■ Solve linear inequalities that require reversing the inequality sign ■ Solve absolute value equations ■ Solve quadratic equations ■ Find solutions to systems of linear equations	■ Interpret and use information from graphs in the coordinate plane ■ Match number line graphs with solution sets of linear inequalities ■ Use the distance formula ■ Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point	■ Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles ■ Use the Pythagorean theorem	■ Use relationships involving area, perimeter, and volume of geometric figures to compute another measure
■ formulate expressions, equations, and inequalities that require planning to accurately model real-world problems (e.g., direct and inverse variation)	■ solve and graph quadratic inequalities	■ make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas ■ investigate angle and arc relationships for circles	■ examine and compare a variety of methods to find areas of composite figures and construct scale drawings

Description of the PLAN Mathematics Test

What Does the PLAN Mathematics Test Measure?

The PLAN Mathematics Test is a 40-question, 40-minute test designed to assess the mathematical reasoning skills that students have typically acquired in many first- and second-year high school courses (pre-algebra, first-year algebra, and plane geometry). While some material from second-year courses is included on the test, most items, including the geometry items, emphasize content presented before the second year of high school. The multiple-choice test requires students to analyze problems in real-world and purely mathematical settings, plan and carry out solution strategies, and verify the appropriateness of solutions. Most of the test questions are individual items, but some may belong to sets (i.e., several items based on the same graph, chart, or information).

On the PLAN Mathematics Test, students demonstrate their ability to read and understand mathematical terminology; to apply definitions, algorithms, theorems, and properties; to interpret and analyze data; and to use mathematics to solve problems.

Students also apply quantitative reasoning in a variety of ways, such as discerning relationships between mathematical concepts, connecting and integrating mathematical concepts and ideas, and making generalizations. Computational skills and knowledge of basic formulas are assumed as background for the problems, but extensive computation and memorization of complex formulas are not required. The concepts covered on the test emphasize the major content areas that are prerequisite to successful performance in upper-level college-prep mathematics courses.

The questions focus on mathematical reasoning and making connections within and among four content areas and at various cognitive levels. These areas and levels are shown in Figure 1 on page 17.

Through the various cognitive levels, students demonstrate their ability to use and reason with mathematics. *Knowledge and Skills* questions (about 35% of the test) require students to use one or more facts, definitions, formulas, or procedures to solve problems that are presented in purely mathematical terms. *Direct Application* questions (about 30% of the test) require students to use their knowledge and skills to solve straightforward problems set in real-world situations. *Understanding Concepts* and *Integrating Conceptual Understanding* questions (about 35% of the test) assess students' depth of understanding of major concepts by requiring reasoning from a single concept or the integration of several concepts to reach an inference or a conclusion.

The content of the PLAN Mathematics Test is reflective of the content taught in mathematics classrooms and of the prerequisite skills and understandings necessary for upper-level college-prep mathematics courses. ACT routinely monitors the

> **"The test should measure what students can do with what they have learned."**
>
> — (ACT, 1996b, p. 2)

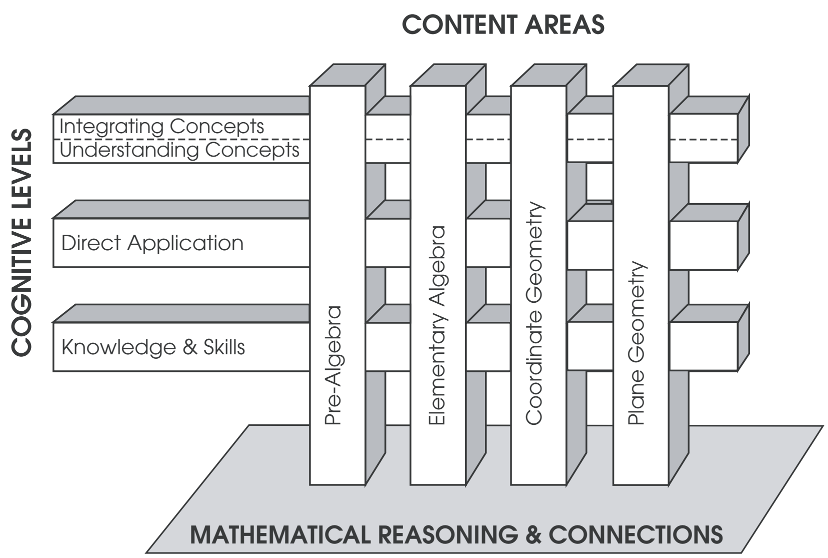
high school mathematics curriculum through reviews of state and national standards, current textbooks, and national organizations' curriculum frameworks; surveys of secondary and postsecondary instructors; and meetings with education consultants. A brief description of the content sampled on the test and the approximate percentage of the test devoted to each content area on the PLAN Mathematics Test are provided below.

Pre-Algebra (35%). Questions in this content area are based on basic operations using whole numbers, decimals, fractions, and integers; place value; square roots and approximations; the concept of exponents; scientific notation; factors; ratio, proportion, and percent; linear equations in one variable; absolute value and ordering numbers by value; elementary counting techniques and simple probability; data collection, representation, and interpretation; and understanding simple descriptive statistics.

Elementary Algebra (20%). Questions in this content area are based on properties of exponents and square roots, evaluation of algebraic expressions through substitution, using variables to express functional relationships, understanding algebraic operations, and solutions of quadratic equations.

Coordinate Geometry (18%). Questions in this content area are based on graphing and the relations between equations and graphs, including points, lines, and parabolas; graphing inequalities; slope; parallel and perpendicular lines; distance; and midpoints.

Plane Geometry (27%). Questions in this content area are based on the properties and relations of plane figures, including angles and relations among perpendicular and parallel lines; properties of circles, triangles, rectangles, parallelograms, and trapezoids; transformations; the concept of proof and proof techniques; volume; and applications of geometry to three dimensions.



Adapted from *Mathematics Framework for the 1996 National Assessment of Educational Progress* (p.11)

Figure 1: **PLAN Mathematics Test Content Areas and Cognitive Levels**

What Is ACT's Calculator Policy for PLAN?

Students may use any four-function, any scientific, and almost any graphing calculator on the PLAN Mathematics Test. However, calculators are **not required.** All problems can be solved without a calculator. If students regularly use a calculator in their math work, they are encouraged to use one they are familiar with as they take the Mathematics Test. Using a more powerful, but unfamiliar, calculator is not likely to give students an advantage over using the kind they normally use.

How Are the Test Questions Linked to the College Readiness Standards?

The PLAN Mathematics Test assesses various kinds and combinations of skills; each of these skills can be measured in different ways. You may have noticed that the strands and the content areas are not the same. The strands are areas in which there are a variety of test questions representing a continuum of skills and understandings. The strands are similar to those found in state and national frameworks. Many of the strands cut across the different content areas on the PLAN Mathematics Test.

Table 2 below provides the strands and the corresponding content areas.

Table 2: **PLAN Mathematics Strands and Corresponding Content Areas**

Strand	**Content Area**
Basic Operations & Applications	Pre-Algebra
Probability, Statistics, & Data Analysis	Pre-Algebra Elementary Algebra
Numbers: Concepts & Properties	Pre-Algebra Elementary Algebra
Expressions, Equations, & Inequalities	Pre-Algebra Elementary Algebra
Graphical Representations	Coordinate Geometry
Properties of Plane Figures	Plane Geometry
Measurement	Plane Geometry Coordinate Geometry

The Need for Thinking Skills

Every student comes to school with the ability to think, but to achieve their goals students need to develop skills such as learning to make new connections between texts and ideas, to understand increasingly complex concepts, and to think through their assumptions. Because of technological advances and the fast pace of our society, it is increasingly important that students not only know information but also know how to critique and manage that information. Students must be provided with the tools for ongoing learning; understanding, analysis, and generalization skills must be developed so that the learner is able to adapt to a variety of situations.

How Are PLAN Test Questions Linked to Thinking Skills?

Our belief in the importance of developing thinking skills in learners was a key factor in the development of PLAN. ACT believes that students' preparation for further learning is best assessed by measuring, as directly as possible, the academic skills that students have acquired and that they will need to perform at the next level of learning. The required academic skills can most directly be assessed by reproducing as faithfully as possible the complexity of the students' schoolwork. Therefore, the PLAN test questions are designed to determine how skillfully students solve problems, grasp implied meanings, draw inferences, evaluate ideas, and make judgments in subject-matter areas important to success in intellectual work both inside and outside school.

Table 3 on pages 20–32 provides sample test questions, organized by score range, that are linked to specific skills within each of the seven Mathematics strands. It is important to note the increasing level of skill with mathematics—computing, reasoning, and making connections—that students scoring in the higher score ranges are able to demonstrate. The questions were chosen to illustrate the variety of content as well as the range of complexity within each strand. The sample test questions for the 13–15, 16–19, 20–23, 24–27, and 28–32 score ranges are the kinds of items answered correctly by 80% or more of the PLAN examinees who obtained scores in each of these five score ranges.

As you review the sample test questions, you will note that each correct answer is marked with an asterisk. For score ranges that include more than one skill, boldface type is used to denote the skill that best corresponds to the sample test question.

"Learning is not attained by chance, it must be sought for with ardour and attended to with diligence."

— Abigail Adams in a letter to
John Quincy Adams

Table 3: **PLAN Sample Test Questions by Score Range**
Basic Operations & Applications Strand

Score Range	***Basic Operations & Applications***	***Sample Test Questions***
13–15	Perform one-operation computation with whole numbers and decimals Solve problems in one or two steps using whole numbers Perform common conversions (e.g., inches to feet or hours to minutes)	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
16–19	Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent **Solve some routine two-step arithmetic problems**	Ten boxes of books were delivered to the school library. There were 50 books in each box, except for the last box, which contained only 40 books. How many books did the library receive in this delivery? **A.** 50 **B.** 450 ***C.** 490 **D.** 500 **E.** 540
20–23	Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	A person goes to the store and purchases the following items: 2 large soft drinks costing \$1.59 each, 1 loaf of bread costing \$1.17, and 8 pounds of potatoes costing 30¢ per pound. If there is a 4% sales tax on all the items, what is the total bill? **A.** \$3.18 **B.** \$5.37 **C.** \$6.75 ***D.** \$7.02 **E.** \$9.45

Table 3: PLAN Sample Test Questions by Score Range
Basic Operations & Applications Strand

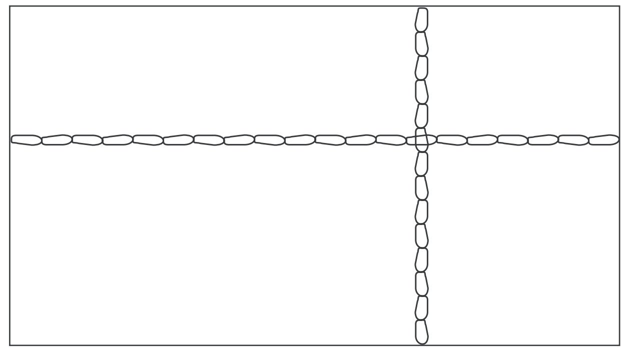
Score Range	***Basic Operations & Applications***	***Sample Test Questions***
24–27	Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	Rachel estimates the dimensions of her rectangular room by walking heel-to-toe between opposite walls, and counting the number of shoe lengths between the walls. Rachel's shoes are each about 9 inches long, and she finds that the dimensions of her room are 14 shoe lengths by 20 shoe lengths (see figure below).  Which of the following is the closest estimate of the dimensions of her room in traditional feet (12 inches)? **A.** 126 by 180 **B.** $18\frac{2}{3}$ by $26\frac{2}{3}$ **C.** 14 by 20 *__D.__ $10\frac{1}{2}$ by 15 **E.** 8 by 10
28–32	Solve word problems containing several rates, proportions, or percentages	In the San Cayetano Elementary School there are 2 music classes of 32 students each. The ratio of boys to girls in the first class is 3:5, and in the second class the ratio of boys to girls is 9:7. What is the ratio of boys to girls if the 2 music classes are combined? **A.** 17:32 **B.** 15:32 *__C.__ 15:17 **D.** 13:19 **E.** 1:1

Table 3: PLAN Sample Test Questions by Score Range
Probability, Statistics, & Data Analysis Strand

<table>
<tr><th>Score Range</th><th>Probability, Statistics, & Data Analysis</th><th>Sample Test Questions</th></tr>
<tr><td>13–15</td><td>Calculate the average of a list of positive whole numbers

Perform a single computation using information from a table or chart</td><td>Kay's Appliance Repair Shop charges for labor according to the chart below. What is the charge for each additional 15 minutes of labor beyond the initial 30 minutes?

<table>
<tr><td>minutes of labor</td><td>1–30</td><td>31–45</td><td>46–60</td><td>61–75</td><td>76–90</td></tr>
<tr><td>charge</td><td>$39.90</td><td>$49.35</td><td>$58.80</td><td>$68.25</td><td>$77.70</td></tr>
</table>

A. $4.73
B. $6.30
C. $7.09
D. $7.56
*E. $9.45</td></tr>
<tr><td>16–19</td><td>Calculate the average of a list of numbers

Calculate the average, given the number of data values and the sum of the data values

Read tables and graphs

Perform computations on data from tables and graphs

Use the relationship between the probability of an event and the probability of its complement</td><td>If the probability that an event will happen is $\frac{5}{8}$, what is the probability that the event will NOT happen?

*A. $\frac{3}{8}$
B. $\frac{3}{5}$
C. $\frac{5}{3}$
D. $\frac{8}{5}$
E. Cannot be determined from the given information</td></tr>
<tr><td>20–23</td><td>Calculate the missing data value, given the average and all data values but one

Translate from one representation of data to another (e.g., a bar graph to a circle graph)

Determine the probability of a simple event

Exhibit knowledge of simple counting techniques</td><td>You need to buy a new notebook at the school supply store. On the shelf the store has 5 red notebooks, 4 blue ones, and 1 green one. If you select 1 of these notebooks at random, what is the probability that you select a blue notebook?

A. $\frac{1}{4}$
*B. $\frac{2}{5}$
C. $\frac{4}{9}$
D. $\frac{4}{5}$
E. 1</td></tr>
</table>

Table 3: **PLAN Sample Test Questions by Score Range**
Probability, Statistics, & Data Analysis Strand

<table>
<tr><th>Score Range</th><th>Probability, Statistics, & Data Analysis</th><th>Sample Test Questions</th></tr>
<tr><td>24–27</td><td>Calculate the average, given the frequency counts of all the data values

Manipulate data from tables and graphs

Compute straightforward probabilities for common situations

Use Venn diagrams in counting</td><td>Mr. Sharifi drew the following frequency table on the chalkboard to show the distribution of grades on a test. What percent of the test grades were C or better?

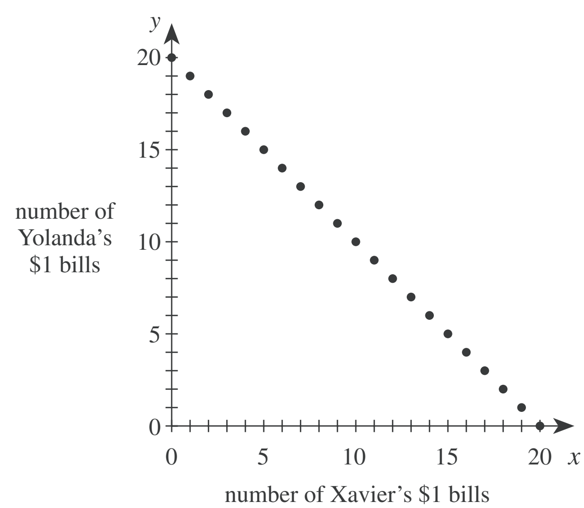
<table>
<tr><th>Grade</th><th>Tally</th><th>Number</th></tr>
<tr><td>A</td><td>||||| |</td><td>6</td></tr>
<tr><td>B</td><td>||||| |||</td><td>8</td></tr>
<tr><td>C</td><td>||||| |||||</td><td>10</td></tr>
<tr><td>D</td><td>||||</td><td>4</td></tr>
<tr><td>F</td><td>||</td><td>2</td></tr>
</table>

A. 24%
B. $33\frac{1}{3}\%$
C. 60%
D. 75%
*E. 80%</td></tr>
<tr><td>28–32</td><td>Calculate or use a weighted average

Interpret and use information from figures, tables, and graphs

Apply counting techniques

Compute a probability when the event and/or sample space are not given or obvious</td><td>Xavier and Yolanda have a total of 20 $1 bills. All of the possible ways to divide the 20 bills between Xavier and Yolanda are graphed below. If Xavier must have an even number of $1 bills, how many possible numbers of $1 bills are there for Yolanda to have?
(Note: Zero is an even number.)



A. 2
B. 10
*C. 11
D. 20
E. 21</td></tr>
</table>

Table 3: **PLAN Sample Test Questions by Score Range**
Numbers: Concepts & Properties Strand

Score Range	***Numbers: Concepts & Properties***	***Sample Test Questions***
13–15	Recognize equivalent fractions and fractions in lowest terms	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
16–19	Recognize one-digit factors of a number **Identify a digit's place value**	In the number 3,859.017 in which place is the "9" ? *__A.__ Ones **B.** Tenths **C.** Tens **D.** Hundredths **E.** Hundreds
20–23	Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor	$\lvert -3(1-4)-2 \rvert = ?$ **A.** -11 **B.** -7 *__C.__ 7 **D.** 11 **E.** 17
24–27	Find and use the least common multiple Order fractions Work with numerical factors Work with scientific notation Work with squares and square roots of numbers **Work problems involving positive integer exponents** Work with cubes and cube roots of numbers Determine when an expression is undefined	For all x and y, $(-2x^2y)^3 = ?$ **A.** $-6x^5y^4$ **B.** $-6x^6y^4$ **C.** $-8x^5y^4$ *__D.__ $-8x^6y^3$ **E.** $-8x^8y^3$
28–32	Apply number properties involving prime factorization Apply number properties involving even/odd numbers and factors/multiples Apply number properties involving positive/negative numbers **Apply rules of exponents**	How many different real values of x satisfy $4^{x+3} = 1$? **A.** None *__B.__ 1 **C.** 2 **D.** 3 **E.** An infinite number

Table 3: **PLAN Sample Test Questions by Score Range**
Expressions, Equations, & Inequalities Strand

Score Range	Expressions, Equations, & Inequalities	Sample Test Questions
13–15	Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) **Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals**	What is the solution of $x + 3.4 = 20.91$? **A.** 24.31 **B.** 23.95 **C.** 17.87 ***D.** 17.51 **E.** 6.15
16–19	**Substitute whole numbers for unknown quantities to evaluate expressions** Solve one-step equations having integer or decimal answers Combine like terms (e.g., $2x + 5x$)	What is the value of $\frac{ab + c}{c}$ when $a = 4$, $b = 0$, and $c = 2$? ***A.** 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5
20–23	Evaluate algebraic expressions by substituting integers for unknown quantities Add and subtract simple algebraic expressions Solve routine first-degree equations **Perform straightforward word-to-symbol translations** Multiply two binomials	Which of the following is a simplified form of $-8x - 4z + 5x + 2y + z - 3y$? **A.** $13x - y + 3z$ **B.** $3x + y + 3z$ **C.** $-3x + 5y - 5z$ **D.** $-3x + y + 3z$ ***E.** $-3x - y - 3z$
24–27	**Solve real-world problems using first-degree equations** Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) Identify solutions to simple quadratic equations Add, subtract, and multiply polynomials Factor simple quadratics (e.g., the difference of squares and perfect square trinomials) Solve first-degree inequalities that do not require reversing the inequality sign	The melting temperatures of 5 metals, in degrees Celsius, are given below: Zinc 420°C Silver 600°C Lead 961°C Gold 1,063°C Tungsten 3,600°C Your Fahrenheit thermometer indicates that a certain metal has a melting point of 1,112° Fahrenheit. Which of the above metals is it most likely to be? (Note: The formula relating temperature in degrees Fahrenheit [F] to degrees Celsius [C] is $F = \frac{9}{5}C + 32$.) **A.** Zinc **B.** Lead ***C.** Silver **D.** Gold **E.** Tungsten

Table 3: **PLAN Sample Test Questions by Score Range**
Expressions, Equations, & Inequalities Strand

Score Range	***Expressions, Equations, & Inequalities***	***Sample Test Questions***
28–32	**Manipulate expressions and equations** Write expressions, equations, and inequalities for common algebra settings Solve linear inequalities that require reversing the inequality sign Solve absolute value equations Solve quadratic equations Find solutions to systems of linear equations	If $A = \frac{1}{2}h(b + c)$, then which of the following is a formula for c in terms of A, b, and h ? **A.** $c = A - h - b$ **B.** $c = 2A - h - b$ **C.** $c = 2A - bh$ **D.** $c = \frac{A}{h} - b$ ***E.** $c = 2\frac{A}{h} - b$

Table 3: **PLAN Sample Test Questions by Score Range**
Graphical Representations Strand

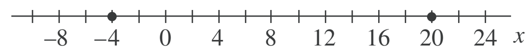
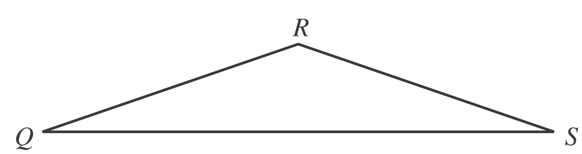
Score Range	***Graphical Representations***	***Sample Test Questions***
13–15	Identify the location of a point with a positive coordinate on the number line	The coordinates of the endpoints of a certain segment on the real number line below are –4 and 20. What is the coordinate of the midpoint of this segment?  **A.** 7 ***B.** 8 **C.** 10 **D.** 12 **E.** 16
16–19	Locate points on the number line and in the first quadrant	In the standard (x,y) coordinate plane, straight line segments are drawn between the following pairs of points: (0,0) and (2,2) (2,2) and (4,0) (4,0) and (2,0) (2,0) and (0,0) What shape is formed by these line segments? ***A.** Triangle **B.** Square **C.** Trapezoid **D.** Pentagon **E.** Hexagon
20–23	Locate points in the coordinate plane Comprehend the concept of length on the number line **Exhibit knowledge of slope**	In the figure below, $\triangle QRS$ is isosceles, $\overline{QS}$ is horizontal, and $\overline{QR} \cong \overline{SR}$. If the slope of $\overline{QR}$ is $\frac{1}{3}$, what is the slope of $\overline{SR}$?  **A.** 3 **B.** $\frac{1}{2}$ ***C.** $-\frac{1}{3}$ **D.** –1 **E.** –3

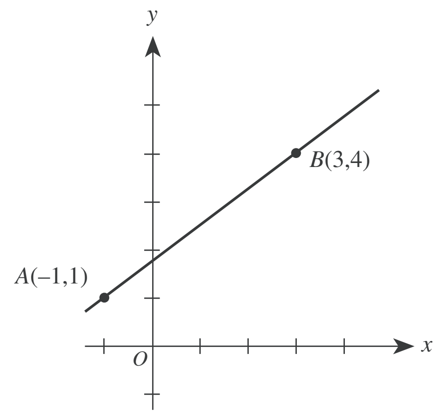
Table 3: **PLAN Sample Test Questions by Score Range** *Graphical Representations* Strand		
Score Range	***Graphical Representations***	***Sample Test Questions***
24–27	Identify the graph of a linear inequality on the number line **Determine the slope of a line from points or equations** Match linear graphs with their equations Find the midpoint of a line segment	The coordinates of points A and B are shown in the standard (x,y) coordinate plane below. What is the slope of $\overleftrightarrow{AB}$?  **A.** $-\frac{3}{4}$ **B.** $\frac{2}{3}$ **C.** $\frac{3}{2}$ ***D.** $\frac{3}{4}$ **E.** $\frac{4}{3}$
28–32	Interpret and use information from graphs in the coordinate plane Match number line graphs with solution sets of linear inequalities Use the distance formula Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.

Table 3: **PLAN Sample Test Questions by Score Range**
Properties of Plane Figures Strand

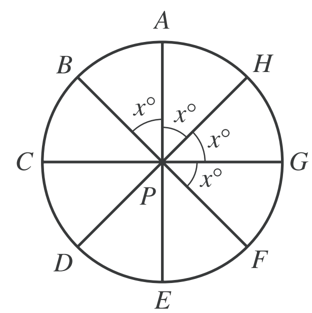
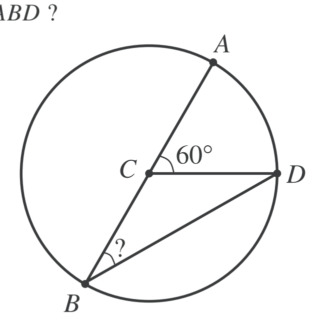
Score Range	***Properties of Plane Figures***	***Sample Test Questions***
13–15		
16–19	Exhibit some knowledge of the angles associated with parallel lines	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
20–23	Find the measure of an angle using properties of parallel lines **Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)**	In the circle below, diameters $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$ intersect at P. The 4 angles marked have the same measure of $x°$. What is the measure of $\angle DPE$?  **A.** $12\frac{1}{2}°$ **B.** $22\frac{1}{2}°$ ***C.** 45° **D.** 72° **E.** 90°
24–27	Use several angle properties to find an unknown angle measure Recognize Pythagorean triples **Use properties of isosceles triangles**	In the circle centered at C below, $\overline{AB}$ is a diameter, and D lies on the circle. If the measure of $\angle ACD$ is 60°, what is the measure of $\angle ABD$?  **A.** 15° ***B.** 30° **C.** 40° **D.** 45° **E.** 60°

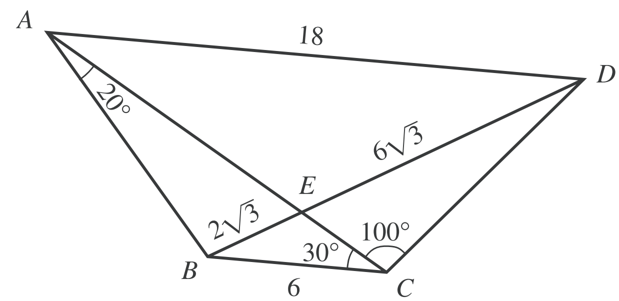
Table 3:	PLAN Sample Test Questions by Score Range *Properties of Plane Figures* Strand	
Score Range	***Properties of Plane Figures***	***Sample Test Questions***
28–32	**Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles** Use the Pythagorean theorem	In trapezoid $ABCD$ pictured below, $\overline{AD}$ is parallel to $\overline{BC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. The measure of $\angle ABC$ is 130°, and the measures of other distances (in centimeters) and angles are as marked. What is the length of diagonal $\overline{AC}$, in centimeters?  **A.** 36 **B.** 12 **C.** $2\sqrt{30}$ **D.** $4\sqrt{3}$ ***E.** $8\sqrt{3}$

Table 3: **PLAN Sample Test Questions by Score Range**
Measurement Strand

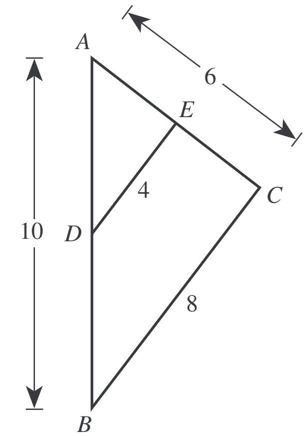
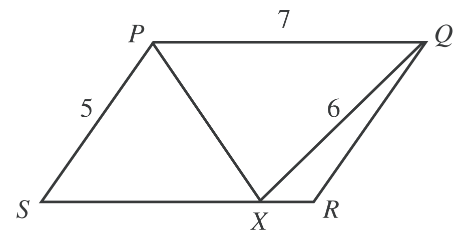
Score Range	***Measurement***	***Sample Test Questions***
13–15	Estimate or calculate the length of a line segment based on other lengths given on a geometric figure	In the figure below, the lengths of line segments are given in feet. If $\overline{BC}$ is parallel to $\overline{DE}$, how many feet long is $\overline{AE}$? **A.** $2\sqrt{21}$ **B.** $2\frac{2}{5}$ ***C.** 3 **D.** $6\frac{2}{3}$ **E.** 12 
16–19	Compute the perimeter of polygons when all side lengths are given **Compute the area of rectangles when whole number dimensions are given**	If rectangle *ABCD* has length 12 inches and width 5 inches, what is its area, in square inches? **A.** 17 **B.** 30 **C.** 34 **D.** 50 ***E.** 60
20–23	**Compute the area and perimeter of triangles and rectangles in simple problems** Use geometric formulas when all necessary information is given	What is the perimeter, in inches, of a square whose sides each measure $5\frac{5}{8}$ inches? **A.** $11\frac{1}{4}$ **B.** $20\frac{5}{8}$ ***C.** $22\frac{1}{2}$ **D.** $25\frac{25}{64}$ **E.** $31\frac{41}{64}$
24–27	Compute the area of triangles and rectangles when one or more additional simple steps are required **Compute the area and circumference of circles after identifying necessary information** Compute the perimeter of simple composite geometric figures with unknown side lengths	What is the area of a circle in the standard (x,y) coordinate plane whose center is (0,0) and whose x-intercepts are (–3,0) and (3,0) ? **A.** 3π **B.** 6π ***C.** 9π **D.** $9\pi^2$ **E.** 36π

Table 3: PLAN Sample Test Questions by Score Range
Measurement Strand

Score Range	***Measurement***	***Sample Test Questions***
28-32	Use relationships involving area, perimeter, and volume of geometric figures to compute another measure	The area of a parallelogram can be found by using the formula $A = bh$, where A is the area, b is the length of the base, and h is the height of the parallelogram. What is the area, in square inches, of $\triangle PQX$ below if the area of parallelogram $PQRS$ is 28 square inches? (Note: Lengths on the diagram are expressed in inches.)  **A.** 21 **B.** 17.5 ***C.** 14 **D.** 13 **E.** 12

Thinking Your Way Through the PLAN Test

In our increasingly complex society, students' ability to think critically and make informed decisions is more important than ever. The workplace demands new skills and knowledge and continual learning; information bombards consumers through media and the Internet; familiar assumptions and values often come into question. More than ever before, students in today's classrooms face a future when they will need to adapt quickly to change, to think about issues in rational and creative ways, to cope with ambiguities, and to find means of applying information to new situations.

Classroom teachers are integrally involved in preparing today's students for their futures. Such preparation must include the development of thinking skills such as problem solving, decision making, and inferential and evaluative thinking. These are, in fact, the types of skills and understandings that underlie the test questions on PLAN.

How Can Analyzing Test Questions Build Thinking Skills?

On pages 34–35, you will find additional sample test questions. The sample test questions provide a link to a strand, a Standard, and a score range. Each sample test question includes a description of the skills and understandings students must demonstrate in order to arrive at the correct answer. The descriptions provide a series of strategies students typically might employ as they work through each test question. Possible flawed strategies leading to the choice of one or more incorrect responses also are offered. Analyzing test questions in this way, as test developers do to produce a Test Question Rationale, can provide students with a means of understanding the knowledge and skills embedded in the test questions and an opportunity to explore why an answer choice is correct or incorrect.

Providing students with strategies such as these encourages them to take charge of their thinking and learning. The sample test questions that appear in Table 3 on pages 20–32 can be used to develop additional Test Question Rationales.

> **"Learning is fundamentally about *making and maintaining connections . . .* among concepts, ideas, and meanings."**
>
> — American Association for Higher Education, American College Personnel Association, & National Association of Student Personnel Administrators, June 1998

Test Question Rationale	
Basic Operations & Applications	■ Calculate the average of a list of positive whole numbers ■ 13–15 score range

1. Mary went bowling with 4 of her friends. Each girl bowled 2 games. These are the results of the games:

	Game 1	Game 2
Mary	120	130
Fran	124	128
June	118	132
Dee	110	142
Linda	120	140

For those 2 games, who had the highest average?

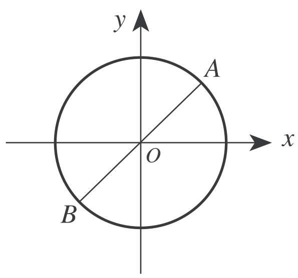
A. Mary
B. Fran
C. June
D. Dee
***E.** Linda

Question 1 is a problem that students may not find difficult, yet it does require some reasoning. Students must decide which numbers to average, based on their reading and understanding of the problem. The formal convention of adding columns of numbers will not lead to the correct answer.

In this problem, there are several opportunities for an insightful student to use shortcuts that will save time and cut down on the possibility of error. One such possibility is to observe that the average is highest when the sum is highest, so it is unnecessary to actually divide all the sums by 2. Another way to simplify the problem is to see that all of the 100s in the girls' scores can be ignored in deciding which average is highest, since everyone is even on that count. Whatever method a student chooses to solve this problem, the result will be that Linda's average is higher than the other averages, and the correct answer is E.

Test Question Rationale	
Graphical Representations	■ Locate points in the coordinate plane ■ 20–23 score range

2. In the figure below, $\overline{AB}$ is a diameter of the circle graphed in the standard coordinate plane. The circle is determined by the equation $x^2 + y^2 = 25$. The (x,y) coordinates of A are (3,4). What are the (x,y) coordinates of B ?



F. (–4,–3)
G. (–4, 3)
***H.** (–3,–4)
J. (–3, 4)
K. (3,–4)

Question 2 is an application of symmetry, and there are many good ways for students to approach the problem. Students who have a basic understanding of the concepts of graphing in coordinate geometry will know that answers G, J, and K may be eliminated because none of them are in the same quadrant as point *B,* which would have negative *x*- and *y*-coordinates. Answer F, which also has negative *x*- and *y*-coordinates, is the most common wrong answer.

Students could decide to solve the problem using the concept of formal slope; or a student might reason that point *A* is as much above the *x*-axis as point *B* is below the *x*-axis, and similarly for the distance right/left of the *y*-axis. Other students might choose to draw the figure to scale in order to check whether the *x*- or the *y*-coordinate was less negative and thus rule out answer F, arriving at the correct answer, H.

From a transformational geometry viewpoint, point *B* can be viewed as a double reflection of point *A,* first about the y-axis and then about the *x*-axis. The first transformation negates the *x*-coordinate and the second negates the *y*-coordinate, giving answer H. Alternatively, point *B* could be viewed as a reflection of point *A* through the origin or as a 180° rotation of point *A* about the origin. (Each of these resulting transformations also maps the circle and the diameter to themselves.)

For a formal slope approach, students might use the following procedures. Points *O, A,* and *B* are on the same line, so the slope of that line can be calculated based on points *O* and *A* as

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{3 - 0} = \frac{4}{3}$. The slope can also be calcu-

lated for points *O* and *B* as $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_B - 0}{x_B - 0} = \frac{y_B}{x_B}$.

The only answer for which $\frac{y}{x} = \frac{4}{3}$ is (–3,–4), where

$\frac{y}{x} = \frac{-4}{-3}$.

The Assessment-Instruction Link

Why Is It Important to Link Assessment With Instruction?

Assessment provides feedback to the learner and the teacher. It bridges the gap between expectations and reality. Assessment can gauge the learners' readiness to extend their knowledge in a given area, measure knowledge gains, identify needs, and determine the learners' ability to transfer what was learned to a new setting.

When teachers use assessment tools to gather information about their students, then modify instruction accordingly, the assessment process becomes an integral part of teaching and learning. Using assessment to inform instruction can help teachers create a successful learning environment.

Students can use assessment as a tool to help them revise and rethink their work, to help integrate prior knowledge with new learning, and to apply their knowledge to new situations. Connecting assessment to classroom instruction can help both teachers and students take charge of thinking and learning.

As teachers review student performances on various measures, they can reexamine how to help students learn. As Peter Airasian, the author of *Classroom Assessment* says, "Assessment is not an end in itself, but a means to another end, namely, good decision making" (p. 19). Linking assessment and instruction prompts both teachers and students to take on new roles and responsibilities. Through reflecting together on their learning, students and teachers can reevaluate their goals and embark on a process of continuous growth.

"*Every* objective, *every* lesson plan, *every* classroom activity, and *every* assessment method should focus on helping students achieve those [significant] outcomes that will help students both in the classroom and beyond."

— Kay Burke, editor of *Authentic Assessment: A Collection*

Are Your Students Developing the Necessary Skills?

Because PLAN is administered during the tenth grade, it allows for a midpoint review of progress students are making in high school. The PLAN results can be used to provide direction for educational and career planning that will allow for adjustment in students' course work to achieve goals after high school. At this stage in their high school careers, students should be encouraged to explore a range of educational and career options based on their current interests and most recent achievements.

EXPLORE and PLAN are developmentally and conceptually linked to the ACT and thus provide a coherent framework for students and counselors and a consistent skills focus for teachers from Grades 8 through 12.

To facilitate the review of students' progress, PLAN and ACT scores are linked through a common score scale and students receive an estimated ACT Composite score along with their PLAN scores. These scores can be used to evaluate students' readiness for college course work and to provide guidance as they prepare for their transition to college or further training. With an ever-increasing number of high school graduates entering college, it becomes the schools' responsibility to ensure that its graduates have mastered the prerequisite skills necessary for success in entry-level courses.

As students and others review test scores from EXPLORE, PLAN, and the ACT, they should be aware that ACT's data clearly reveal that students' ACT test scores are directly related to preparation for college. Students who take rigorous high school courses, which ACT has defined as core college preparatory courses, achieve much higher test scores than students who do not. ACT has defined core college preparatory course work as four or more years of English, and three or more years each of mathematics, social studies, and natural science.

ACT works with colleges to help them develop guidelines that place students in courses that are appropriate for their level of achievement as measured by the ACT. In doing this work, ACT has gathered course grade and test score data from a large number of first-year students across a wide range of postsecondary institutions. These data provide an overall measure of what it takes to be successful in a standard first-year college course. Data from 98 institutions and over 90,000 students were used to establish the ACT College Readiness Benchmark Scores, which are median course placement scores achieved on the ACT that are directly reflective of student success in a college course.

Success is defined as a 50 percent chance that a student will earn a grade of B or better. The courses are the ones most commonly taken by first-year students in the areas of English, mathematics, social studies, and science, namely English Composition, College Algebra, an entry-level College Social Studies/Humanities course, and College Biology. The ACT scores established as the ACT College Readiness Benchmark Scores are 18 on the English Test, 22 on the Mathematics Test, 21 on the Reading Test, and 24 on the Science Test. The College Readiness Benchmark Scores were based upon a sample of postsecondary institutions from across the United States. The data from these institutions were weighted to reflect postsecondary institutions nationally. The Benchmark Scores are median course placement values for these institutions and as such represent a *typical* set of expectations.

College Readiness Benchmark Scores have also been developed for EXPLORE and for PLAN, to indicate a student's probable readiness for college-level work, in the same courses named above, by the time the student graduates from high school. The EXPLORE and PLAN College Readiness Benchmark Scores were developed using records of students who had taken EXPLORE, PLAN, and the ACT (four years of matched data). Using either EXPLORE subject-area scores or PLAN subject-area scores, we estimated the conditional probabilities associated with meeting or exceeding the corresponding ACT Benchmark Score. Thus, each EXPLORE (1–25) or PLAN (1–32) score was associated with an estimated probability of meeting or exceeding the relevant ACT Benchmark Score. We then identified the EXPLORE and PLAN scores, at Grades 8, 9, 10, and 11, that came the closest to a 0.5 probability of meeting or exceeding the ACT Benchmark Score, by subject area. These scores were selected as the EXPLORE and PLAN Benchmark Scores.

All the Benchmark Scores are given in Table 4. Note that, for example, the first row of the table should be read as follows: An eighth-grade student who scores 13, or a ninth-grade student who scores 14, on the EXPLORE English Test has a 50 percent probability of scoring 18 on the ACT English Test; and a tenth-grade student who scores 15, or an eleventh-grade student who scores 17, on the PLAN English Test has a 50 percent probability of scoring 18 on the ACT English Test.

Table 4: **College Readiness Benchmark Scores**

Subject Test	EXPLORE Test Score Grade 8	EXPLORE Test Score Grade 9	PLAN Test Score Grade 10	PLAN Test Score Grade 11	ACT Test Score
English	13	14	15	17	18
Mathematics	17	18	19	21	22
Reading	15	16	17	19	21
Science	20	20	21	23	24

Using Assessment Information to Help Support Low-Scoring Students

Students who receive a Composite score of 16 or below on PLAN will most likely require additional guidance and support from their teachers and family in order to meet their academic goals, particularly if one of those goals is to attend a four-year college or university.

College admission policies vary widely in their level of selectivity. ACT Composite scores typically required by colleges having varying levels of selectivity are shown in Table 5. This information provides only general guidelines. There is considerable overlap among admission categories, and colleges often make exceptions to their stated admission policies.

Table 5: **The Link Between ACT Composite Scores and College Admission Policies**

Admission Policy	Typical Class Rank of Admitted Students	Typical ACT Composite Scores of Admitted Students
Highly Selective	Majority of accepted freshmen in top 10% of high school graduating class	25–30
Selective	Majority of accepted freshmen in top 25% of high school graduating class	21–26
Traditional	Majority of accepted freshmen in top 50% of high school graduating class	18–24
Liberal	Some of accepted freshmen from lower half of high school graduating class	17–22
Open	All high school graduates accepted to limit of capacity	16–21

A student's PLAN Composite score is one indicator of the student's readiness for college-level work. For each student's PLAN Composite score, an estimated ACT score range is reported. The estimated ACT Composite score range refers to the score a student would be expected to obtain in the fall of his or her senior year. The estimated fall twelfth-grade score ranges for students who take PLAN in the fall of tenth grade are reported in Table 6.

Table 6 indicates that, for a PLAN Composite score of 13 in fall of tenth grade, the lower limit of the estimated fall twelfth-grade ACT Composite score range is given as 13 and the upper limit is given as 17. That is, an estimated ACT Composite score range of 13 to 17 is reported for students who receive PLAN Composite scores of 13 when tested in the fall of tenth grade.

In interpreting the estimated ACT Composite score ranges, it's important to note that EXPLORE, PLAN, and the ACT are curriculum-based testing programs. This is one reason ACT expects that some students will fall short of or improve upon their estimated ACT score ranges. If students do not maintain good academic work in high school, their actual ACT Composite scores may fall short of their estimated score ranges. The converse is also true; some students who improve their academic performance may earn ACT Composite scores higher than estimated.

As students review their PLAN test scores, they should be encouraged to think about their postsecondary education or training plans. Test scores should be discussed in the context of students' future goals, previous academic preparation, and plans for future high school course work. As educators and parents look over students' content-area test scores, the way students' scores match up with their goals will become clear. For example, a student who wishes to become an engineer will need a solid mathematics background. A high Mathematics Test score can be used as evidence that the goal is realistic. A low score (or subscore) suggests the student should consider ways of improving his or her scientific knowledge and skills through additional course work and/or added effort in the area.

Table 6: **Estimated ACT Composite Score Ranges**

PLAN Composite Score	**Estimated ACT Composite Score Range**	
	Low Score	**High Score**
1	8	10
2	8	10
3	8	10
4	8	11
5	8	11
6	9	12
7	10	13
8	11	14
9	11	14
10	11	15
11	12	15
12	13	17
13	13	17
14	14	18
15	15	19
16	16	20
17	17	21
18	19	23
19	20	24
20	21	25
21	22	26
22	23	27
23	24	28
24	26	30
25	26	30
26	27	31
27	28	32
28	29	33
29	30	33
30	31	34
31	33	35
32	33	35

Eighth or ninth grade is a good time to begin taking demanding course work (Noeth & Wimberly, 2002). “Many studies have found . . . that taking high-level math courses increases college-going among minority and first generation college students. Further, students who take algebra in eighth grade are very likely to apply to a four-year college, controlling for other high school course taking” (Noeth & Wimberly, 2002, p. 17).

In addition to planning for high school course work, taking remedial classes if necessary, and beginning to match career goals to known talents, tenth-grade students who want to attend a four-year college or university should begin educating themselves about such schools. Some students, particularly those whose parents did not attend college, may not have access to information about postsecondary education. “Though many students . . . attending urban schools may have the desire and expectation, they may not have the skills, knowledge, and information they need to enter and complete a postsecondary program. Many . . . do not have the informational resources, personal support networks, continual checkpoints, or structured programs to make college exploration and planning a theme throughout their daily lives. . . . Students need their schools, parents, and others to help them plan for college and their future careers” (Noeth & Wimberly, 2002, p. 4).

What Does It Mean to Be a Low-Scoring Student?

Low-achieving students tend to be those students who score low on standardized tests. Students who slip behind are the likeliest to drop out and least likely to overcome social and personal disadvantages.

According to Judson Hixson, a researcher at the North Central Regional Educational Laboratory (NCREL), students who are at risk should be considered in a new light:

> Students are placed “at risk” when they experience a significant mismatch between their circumstances and needs, and the capacity or willingness of the school to accept, accommodate, and respond to them in a manner that supports and enables their maximum social, emotional, and intellectual growth and development.
>
> As the degree of mismatch increases, so does the likelihood that they will fail to either complete their elementary and secondary education, or more importantly, to benefit from it in a manner that ensures they have the knowledge, skills, and dispositions necessary to be successful in the next stage of their lives—that is, to successfully pursue postsecondary education, training, or meaningful employment and to participate in, and contribute to, the social, economic, and political life of their community and society as a whole.
>
> The focus of our efforts, therefore, should be on enhancing our institutional and professional capacity and responsiveness, rather than categorizing and penalizing students for simply being who they are. (Hixson, 1993, p. 2)

Hixson’s views reveal the necessity of looking at all the variables that could affect students’ performance, not just focusing on the students themselves.

Low-achieving students may demonstrate some of the following characteristics:

- difficulty with the volume of work to be completed;
- low reading and writing skills;
- low motivation;
- low self-esteem;
- poor study habits;
- lack of concentration;
- reluctance to participate in class or to ask for help with tasks/assignments; and
- test anxiety.

Many of these characteristics are interconnected. For example, a low-scoring student cannot complete the volume of work a successful student can if it takes a much longer time for that low-scoring student to decipher text passages because of low reading skills. There is also the issue of intrinsic motivation: students may have little desire to keep trying if they do not habitually experience success.

Some low-scoring students may not lack motivation or good study habits, but may still be in the process of learning English; still others may have learning disabilities that make it difficult for them to do complex work in one or two content areas.

Again, we must not focus only on the students themselves, but also consider other variables that could affect their academic performance, such as

- job or home responsibilities that take time away from school responsibilities;
- parental attitude toward and involvement in students' school success;
- students' relationships with their peers;
- lack of adequate support and resources; and
- lack of opportunities.

For example, some students who score low on tests are never introduced to a curriculum that challenges them or that addresses their particular needs: "Much of the student stratification within academic courses reflects the social and economic stratification of society. Schools using tracking systems or other methods that ultimately place low-income and marginal students in lower-level academic courses are not adequately preparing them to plan for postsecondary education, succeed in college, and prepare for lifelong learning" (Noeth & Wimberly, 2002, p. 18).

As Barbara Means and Michael Knapp have suggested, many schools need to reconstruct their curricula, employing instructional strategies that help students to understand how experts think through problems or tasks, to discover multiple ways to solve a problem, to complete complex tasks by receiving support (e.g., cues, modifications), and to engage actively in classroom discussions (1991).

Many individuals and organizations are interested in helping students succeed in the classroom and in the future. For example, the Network for Equity in Student Achievement (NESA), a group of large urban school systems, and the Minority Student Achievement Network (MSAN), a group of school districts in diverse suburban areas and small cities, are organizations that are dedicated to initiating strategies that will close the achievement gap among groups of students. Many schools and districts have found participation in such consortia to be helpful.

According to Michael Sadowski, editor of the *Harvard Education Letter,* administrators and teachers who are frustrated by persistent achievement gaps within their school districts "have started to look for answers within the walls of their own schools. They're studying school records, disaggregating test score and grade data, interviewing students and teachers, administrating questionnaires—essentially becoming researchers—to identify exactly where problems exist and to design solutions" (Sadowski, 2001, p. 1).

A student may get a low score on a standardized test for any of a number of reasons. To reduce the probability of that outcome, the following pages provide information about factors that affect student performance as well as some suggestions about what educators and students can do before students' achievement is assessed on standardized tests like PLAN.

What Are Some Factors That Affect Student Performance?

Many factors affect student achievement. Diane Ravitch, a research professor at New York University, has identified several positive factors in her book *The Schools We Deserve: Reflections on the Educational Crisis of Our Time* (1985, pp. 276 and 294). These factors, which were common to those schools that were considered effective in teaching students, include

- a principal who has a clearly articulated vision for the school, and the leadership skills to empower teachers to work toward that vision;
- a strong, clearly thought-out curriculum in which knowledge gained in one grade is built upon in the next;
- dedicated educators working in their field of expertise;
- school-wide commitment to learning, to becoming a "community of learners";
- a blend of students from diverse backgrounds;
- "high expectations for all" students; and
- systematic monitoring of student progress through an assessment system.

There are also factors that have a negative impact on student achievement. For example, some students "may not know about, know how, or feel entitled to take academic advantage of certain opportunities, like college preparatory courses, college entrance exams, and extracurricular learning opportunities" (Goodwin, 2000, p. 3).

All students need to be motivated to perform well academically, and they need informed guidance in sorting out their educational/career aspirations. Teachers who challenge their students by providing a curriculum that is rigorous and relevant to their world and needs (Brewer, Rees, & Argys, 1995; Gay, 2000), and who have a degree and certification in the area in which they teach (Ingersoll, 1998) and ample opportunities to collaborate with their peers (McCollum, 2000), are more likely to engender students' success in school.

Making the Invisible Visible

Using assessment information, such as that provided by the EXPLORE, PLAN, and ACT tests in ACT's Educational Planning and Assessment System (EPAS), can help bring into view factors that may affect—either positively or negatively—student performance. Reviewing and interpreting assessment information can encourage conversations between parents and teachers about what is best for students. Using data is one way of making the assumptions you have about your students and school, or the needs of students, visible.

Collecting assessment information in a systematic way can help teachers in various ways. It can help teachers see more clearly what is happening in their classrooms, provide evidence that the method of teaching they're using really works, and determine what is most important to do next. As teachers become active teacher-researchers, they can gain a sense of control and efficacy that contributes to their sense of accomplishment about what they do each day.

There are many different types of assessment information that a school or school district can collect. Some types yield quantitative data (performance described in numerical terms), others qualitative data (performance described in nonnumerical terms, such as text, audio, video, or photographs). All types, when properly analyzed, can yield useful insights into student learning. For example, schools and teachers can collect information from

- standardized tests (norm- or criterion-referenced tests);
- performance assessments (such as portfolios, projects, artifacts, presentations);
- peer assessments;
- progress reports (qualitative, quantitative, or both) on student skills and outcomes;
- self-reports, logs, journals; and
- rubrics and rating scales.

Reviewing student learning information in the context of demographic data may also provide insight and information about specific groups of students, like low-scoring students. Schools therefore would benefit by collecting data about

- enrollment, mobility, and housing trends;
- staff and student attendance rates and tardiness rates;
- dropout, retention, and graduation rates;
- gender, race, ethnicity, and health;
- percent of free/reduced lunch and/or public assistance;
- level of language proficiency;
- staff/student ratios;
- number of courses taught by teachers outside their endorsed content area;
- retirement projections and turnover rates; and
- teaching and student awards.

What Can Educators and Students Do Before Students Take Standardized Tests?

Integrate assessment and instruction. Because PLAN is curriculum-based, the most important prerequisite for optimum performance on the test is a sound, comprehensive educational program. This "preparation" begins long before any test date. Judith Langer, the director of the National Research Center on English Learning and Achievement, conducted a five-year study that compared the English programs of typical schools to those that get outstanding results. Schools with economically disadvantaged and diverse student populations in California, Florida, New York, and Texas predominated the study. Langer's study revealed that in higher performing schools "test preparation has been integrated into the class time, as part of the ongoing English language arts learning goals." This means that teachers discuss the demands of high-stakes tests and how they "relate to district and state standards and expectations as well as to their curriculum" (Langer, 2000, p. 6).

Emphasize core courses. ACT research conducted in urban schools both in 1998 and 1999 shows that urban school students can substantially improve their readiness for college by taking a more demanding sequence of core academic courses in high school. Urban students taking a more rigorous sequence of courses in mathematics and science and finding success in those courses score at or above national averages on the ACT. Regardless of gender, ethnicity, or family income, those students who elect to take four or more years of rigorous English courses and three or more years of rigorous course work in mathematics, science, and social studies earn higher ACT scores and are more successful in college than those who have not taken those courses (ACT & Council of Great City Schools, 1999). Subsequent research has substantiated these findings and confirmed the value of rigor in the core courses (ACT, 2004; ACT & The Education Trust, 2004).

Teach test-taking strategies. Students may be helped by being taught specific test-taking strategies, such as the following:

- Learn to pace yourself.
- Know the directions and understand the answer sheet.
- Read carefully and thoroughly.
- Answer easier questions first; skip harder questions and return to them later.
- Review answers and check work, if time allows.
- Mark the answer sheet quickly and neatly; avoid erasure marks on the answer sheet.
- Answer every question (you are not penalized for guessing on PLAN).
- Become familiar with test administration procedures.
- Read all the answer choices before you decide which is the correct answer.

Students are more likely to perform at their best on a test if they are comfortable with the test format, know appropriate test-taking strategies, and are aware of the test administration procedures. Test preparation activities that help students perform better in the short term will be helpful to those students who have little experience taking standardized tests or who are unfamiliar with the format of PLAN.

What Do the PLAN Mathematics Test Results Indicate About Low-Scoring Students?

Students who score below 16 on the PLAN Mathematics Test are likely to have some or all of the knowledge and skills described in the PLAN Mathematics College Readiness Standards for the 13–15 range. In fact, they may well have some of the skills listed in the 16–19 range. Low-scoring students may be able to demonstrate skills in a classroom setting that they are not able to demonstrate in a testing situation. Therefore, these students need to become more consistent in demonstrating these skills in a variety of contexts and situations.

The EPAS Mathematics College Readiness Standards indicate that students who score below 16 tend to have the ability to

- Perform one-operation computation with whole numbers and decimals
- Solve problems in one or two steps using whole numbers
- Perform common conversions (e.g., inches to feet or hours to minutes)
- Calculate the average of a list of positive whole numbers
- Perform a single computation using information from a table or chart
- Recognize equivalent fractions and fractions in lowest terms
- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals
- Identify the location of a point with a positive coordinate on the number line
- Estimate or calculate the length of a line segment based on other lengths given on a geometric figure

In sum, these students typically show skill working primarily with whole numbers and decimals, whether they are looking at data, solving equations, or dealing with measuring geometric figures. These students will likely benefit from encouragement in performing calculations and solving equations involving rational numbers; in extending their knowledge of graphing to the coordinate plane; in becoming more comfortable with the basic concepts of probability, statistics, and data analysis through real-world problems; and in extending measurement concepts to include perimeter and area for a variety of geometric figures.

What Does Research Say About How Mathematics Instruction Should be Conducted?

Research suggests that learning is maximized when students take a demanding core curriculum and engage in rigorous learning activities. The core curriculum must be embedded in a learning environment where students are motivated to work hard. To be motivated, students need to see the relevance of their schoolwork (LaPoint, Jordan, McPartland, & Towns, 1996). Research also suggests that framing learning and performance tasks within contexts that are familiar cultural experiences may improve students' cognitive functioning and consequently their achievement (Boykin & Bailey, 2000).

National Council of Teachers of Mathematics (NCTM) recommendations for change in mathematics education call for teachers to use a wide range of instructional strategies:

> A variety of instructional methods should be used in classrooms to cultivate students' abilities to investigate, make sense of, and construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instruction, and whole-class discussions in which the teacher serves as a moderator. (NCTM, 1989, pp. 125, 128)

What Can Be Done to Help Students Understand the Mathematical Situations They Encounter?

Students need to see how mathematics crosses into other disciplines and is used in real life, not just as isolated pieces of information to be used only in mathematics class. Today's mathematics instruction should not be just about memorization, though some things like the basic facts of arithmetic need to be internalized to the point of automatic recall. The memorization-and-drill approach does not provide an in-depth understanding of those mathematical concepts required to build a strong foundation in math, nor does the lecture method.

The "scaffolding" method, in which the teacher sets up interactive learning activities at increasingly challenging levels as students progress toward mastery of a concept, can be used as an alternative to drills and lecture. The learning activities can be organized so that the students are interacting with the teacher or with their peers. The teacher needs to monitor group discussions and guide the students as they use strategies of questioning, summarizing, clarifying, and predicting to achieve comprehension of the major concept(s) being taught.

For example, a teacher could increase the level of difficulty in algebraic equations by using a larger coefficient of the variable. Another idea would be to have students who have internalized how to solve algebraic equations work with others who still need assistance, by playing a game of "concentration." The teacher could make a set of cards with simple one-step equations and their solutions and have the students pair a one-step equation with the correct solution as a "match." The level of difficulty for the game could be increased by making another set of cards that includes a variety of one- and two-step equations. Each student takes a turn selecting cards until he or she gets a matched pair (one-step equations) or matched group (two-step equations). Play would continue until all of the cards were selected, and a player's standing within the group could be calculated by assigning points for each "match." Matched groups that involve two steps for their solution could be worth more points.

Mathematics study should include investigation of patterns in numbers, shapes, data, probability, and growth/decay, along with practice in communicating mathematically. Students should also be encouraged to guess:

> Part of developing student confidence is the realization that guessing plays an important role in learning mathematics. Even so, students must learn that the guesses must be tempered by validation and attempts to structure an explanation as to why the guess is an appropriate response to the situation at hand. Developing the ability to conjecture, test, revise, and reconjecture is an important step toward thinking mathematically. (Association for Supervision and Curriculum Development [ASCD], 1999, p. 9)

Today, mathematics requires students to move beyond memorization and drill and to develop skills that include problem solving, reasoning, representation, and communication. Students need to be able to visualize and make connections between the various branches of mathematics and between mathematics and other content areas. Students might need to solve a probability problem geometrically by using coordinates from a Cartesian plane, or to use their knowledge of number concepts, properties, and theorems (such as exponentiation, root extraction, and the Pythagorean theorem) along with their algebraic manipulation skills to solve a measurement problem involving perimeter, area, or volume of a composite geometric figure. Teachers need to guide their students through activities such as small-group work, individual explorations, use of technology, peer instruction, and math logs/journals so that the students have the opportunity to develop reasoning and problem-solving skills—skills they'll need to use when confronted with purely mathematical problems and in real-world situations with connections to other content areas.

Teachers also need to help students understand difficult or abstract concepts by using real-life examples to make the concepts more concrete. For example, the order of operations to solve two-step algebra equations could be reinforced using shoes and socks. When students get dressed they put on socks first, then shoes; but when they want to undress, they do the reverse: they remove instead of put on, removing their shoes first and socks second. In analyzing the equation $2x + 3 = 10$, the order of operations is to multiply x by 2 and then add 3. To solve the equation, the inverse operations are used in the reverse order: subtract 3 and then divide by 2.

Analyze the equation	→	get dressed
Multiply x by 2	→	put on socks
Add 3	→	put on shoes
Solve the equation	→	get undressed
Subtract 3	→	take off shoes
Divide by 2	→	take off socks

By connecting the abstract to the concrete as in the shoes and socks example, the teacher can reinforce mathematical concepts by relating them to situations in everyday life.

What Knowledge and Skills Are Low-Scoring Students Ready to Learn?

For students who score below 16 on the PLAN Mathematics Test, their target achievement outcomes could be the College Readiness Standards listed in the 16–19 range:

- Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent
- Solve some routine two-step arithmetic problems
- Calculate the average of a list of numbers
- Calculate the average, given the number of data values and the sum of the data values
- Read tables and graphs
- Perform computations on data from tables and graphs
- Use the relationship between the probability of an event and the probability of its complement
- Recognize one-digit factors of a number
- Identify a digit's place value
- Substitute whole numbers for unknown quantities to evaluate expressions
- Solve one-step equations having integer or decimal answers
- Combine like terms (e.g., $2x + 5x$)

- Locate points on the number line and in the first quadrant
- Exhibit some knowledge of the angles associated with parallel lines
- Compute the perimeter of polygons when all side lengths are given
- Compute the area of rectangles when whole number dimensions are given

By no means should these be seen as limiting or exclusive goals. As stated earlier, it is important to use multiple sources of information to make instructional decisions and to recognize that individual students learn at different rates and in different sequences. What's important is to get students communicating mathematically.

What Strategies/Materials Can Teachers Use in Their Classrooms?

According to Bryan Goodwin, senior program associate at the Mid-continent Research Education Laboratory (McREL), "it is important to note that improving the performance of disenfranchised students does not mean ignoring other students. Indeed, many of the changes advocated—such as making curricula more rigorous and creating smaller school units—will benefit all students" (Goodwin, 2000, p. 6). Means and Knapp (1991) express a similar view:

> A fundamental assumption underlying much of the curriculum in America's schools is that certain skills are "basic" and must be mastered before students receive instruction on more "advanced" skills, such as reading comprehension, written composition, and mathematical reasoning. . . . Research from cognitive science questions this assumption and leads to a quite different view of children's learning and appropriate instruction. By discarding assumptions about skill hierarchies and attempting to understand children's competencies as constructed and evolving both inside and outside of school, researchers are developing models of intervention that start with what children know and provide access to explicit models of thinking in areas that traditionally have been termed "advanced" or "higher order." (p. 1)

Pages 50–56 exemplify the kind of teacher-developed activity that could be used in a classroom for all students, not just those who have scored low on a standardized assessment like PLAN. The activity, which has two parts, has students compute perimeter and area of rectangles. The students are then asked to express a relationship that they discovered in their own words (using appropriate mathematical language) and algebraically (using precise mathematical notation). A Project Worksheet is included that could be used by the teacher to assess student learning. A simple Rubric is included, along with a Group Questionnaire and Quiz as suggestions for other types of assessments that could be used to evaluate students' learning. The activity provides suggestions for related investigations that would enable the teacher to extend and build on the original ideas in order to reinforce and strengthen student learning.

How Is the Activity Organized?

A template for the instructional activity appears on page 49. Since the instructional activity has multiple components, an explanation of each is provided below.

A The primary *Mathematics Strands* are displayed across the top of the page. The strand name "Probability, Statistics, & Data Analysis" is abbreviated to "Data Analysis."

B The *Guiding Principles* section consists of one or more statements about instruction, assessment, thinking skills, student learning, and other educationally relevant topics.

C The *Title and Subject Area(s)/Course(s)* information allows you to determine at a glance the primary focus of the activity and whether it might meet the needs of your student population.

D The *Purpose* statement describes knowledge and skills students may have difficulty with and what will be done in the activity to help them acquire that knowledge and skills.

E The *Overview* section provides a brief description of how the knowledge and skills listed in the purpose statement will be taught and suggests an estimated time frame for the entire activity.

F The *Links to College Readiness Standards* section indicates the primary knowledge and skills the activity will focus on. These statements are tied directly to the strands listed at the top of the page.

G The next section, *Description of the Instructional Activity,* is divided into three interrelated parts: Materials/Resources, Introduction, and Suggested Teaching Strategies/Procedures. The section provides suggestions for engaging students in the activity, and gives related topics and tasks. The activity addresses a range of objectives and modes of instruction, but it emphasizes providing students with experiences that focus on reasoning and making connections, use community resources and real-life learning techniques, and encourage students to ask questions—questions leading to analysis, reflection, and further study and to individual construction of meanings and interpretations.

H Valuable *Comments/Tips from Classroom Teachers* are provided for the activity. As the title indicates, this text box includes ideas from current classroom teachers.

I The *Suggestions for Assessment* section offers ideas for documenting and recording student learning. This section describes two types of assessments: Embedded Assessments and Summative Assessments. Embedded Assessments are assessments that inform you as to where your students currently are in the learning process (a formative assessment that is primarily teacher developed and is integral to the instructional process—at times the instruction and assessment are indistinguishable). The second type of assessment is a Summative Assessment (a final assessment of students' learning), which provides a description of the knowledge and skills students are to have mastered by the end of the activity and the criteria by which they will be assessed.

J The *Links to Ideas for Progress* section provides statements that suggest learning experiences (knowledge and skills to be developed) that are connected to the Suggested Strategies/Activities.

K The *Suggested Strategies/Activities* section provides a brief description of ways to reteach the skills or content previously taught or to extend students' learning

This teacher-developed activity provides suggestions, not prescriptions. You are the best judge of what is necessary and relevant for your students. Therefore, we encourage you to review the activity, modifying and using those suggestions that apply, and disregarding those that are not appropriate for your students. As you select, modify, and revise the activity, you can be guided by the statements that appear in the Guiding Principles box at the beginning of the activity.

Linking Instruction and Assessment

Strand(s): A

Guiding Principles

- B
-
-

TITLE

C

Subject Area(s)/Course(s)

Purpose D

Overview E

Links to College Readiness Standards

- F
-
-

Description of the Instructional Activity

Materials/Resources

- G
-
-

Introduction—

Suggested Teaching Strategies/Procedures—

Comments/Tips from Classroom Teachers:

H

Suggestions for Assessment I

Embedded Assessment (name of assessment)—

Embedded Assessment (name of assessment)—

Summative Assessment (name of assessment)—

ENHANCING STUDENT LEARNING

Links to Ideas for Progress J

-
-
-

Suggested Strategies/Activities K

Guiding Principles

- "Classroom strategies have a strong focus on developing thinking skills in a systematic fashion that elevates the importance of critical thinking, problem solving, reasoning, and creative thinking over less complex intellectual tasks, such as recall of factual information." (Association for Supervision and Curriculum Development [ASCD], 1994, p. 2.33)
- "Understanding develops as students construct new relationships among ideas, as they strengthen existing relationships among those ideas, and as they reorganize their ideas." (Secada, 1997, p. 8)
- "To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption." (National Council of Teachers of Mathematics [NCTM], 2000, p. 23)

Comparing Area and Perimeter

Algebra I and Geometry

Purpose

Finding area and perimeter is difficult for many students. This activity gives students useful practice in measuring, calculating, and comparing the areas and perimeters of different rectangles.

Overview

After introducing or reviewing the area and perimeter of rectangles and the concept of congruent and noncongruent rectangles, divide students into small groups and have them measure the sides of a given sheet of paper and calculate the area and perimeter. They will then cut the paper into 4 smaller rectangles. The students will measure the sides of each of the smaller rectangles and calculate the areas and perimeters of each. Students will then find the sum of the 4 areas and the sum of the 4 perimeters and compare these values to the original area and perimeter. The approximate time for this activity is 50 minutes.

Links to College Readiness Standards

- Compute the perimeter of polygons when all side lengths are given
- Compute the area of rectangles when whole number dimensions are given

Description of the Instructional Activity

Materials/Resources

- Calculator (optional)
- Construction paper
- Pencil
- Ruler
- Scissors
- Project Worksheet (p. 53)
- Optional Assessments:
  - ✓ Group Questionnaire (p. 54)
  - ✓ Rubric for the Project Worksheet (p. 55)
  - ✓ Quiz (p. 56)

Introduction—Introduce or review the concepts of area (in square units) and perimeter (in units) of rectangles. Discuss real-life situations in which you would need to calculate area versus perimeter and vice versa. Using a whiteboard or overhead projector, provide some examples where area and perimeter are calculated (e.g., carpeting, fencing, painting, trim). Use the formulas $A = lw$ and $P = 2(l + w)$ or $P = 2l + 2w$, where:

- A represents the area, in square units, of the rectangle;
- P represents the perimeter, in units, of the rectangle;
- l represents the length, in units, of the rectangle; and
- w represents the width, in units, of the rectangle.

Introduce or review the concepts of congruent and noncongruent rectangles. Explain that congruent rectangles are 2 or more rectangles that have the same length and width, although the orientation of the rectangles does not have to be the same.

Suggested Teaching Strategies/Procedures—Prepare sheets of construction paper for the class. Leave some sheets as 9-inch-by-12-inch rectangles. Cut other sheets into a variety of sizes of rectangles (e.g., 6" x 8", 8" x 11", 9" x 9", 9" x 10").

Divide the class into groups of 3–5 students. Each student in each group will receive a sheet of construction paper, a ruler, a pair of scissors, and a Project Worksheet that can be found on page 53. The sheets of construction paper distributed to each student within each group should be of a different size, with one of the sheets being a square.

Have each student measure all 4 sides of his or her original rectangle and record the length (l) and width (w) in Table 1 on the Project Worksheet. Then, each student should calculate (by hand) the area (A) and perimeter (P) of his or her original rectangle and record these values to finish the first row in Table 1 of the Project Worksheet. Each student should then check the answers with a calculator (optional) or have another member of the group check the answers.

Next, instruct each student to cut the original rectangle into 4 smaller rectangles (students must use the whole sheet of paper). One member of each group should be instructed to cut the original rectangle into 4 congruent rectangles while the other members should cut their original rectangle into 4 noncongruent rectangles. If possible, make sure that in at least one group, the person who had the square as an original rectangle will cut the square into congruent rectangles and that in at least one group, the person who had the square as an original rectangle will cut the square into noncongruent rectangles. To ensure that at least one square gets cut into four congruent pieces and at least one square gets cut into four noncongruent pieces, the teacher could provide pre-marked sheets of construction paper.

Comments/Tips from Classroom Teachers:

Pre-marked sheets would guarantee that students only work with whole numbers and won't need to be concerned with choosing a degree of accuracy for decimal or fraction measures.

Each student should number the 4 smaller rectangles 1–4. Next, each student should measure all 4 sides of each of the 4 smaller rectangles and record the length (l_1–l_4) and width (w_1–w_4) of each in Table 1 of the Project Worksheet (these measurements may be decimals or fractions). Then, he or she should calculate the area (A_1–A_4) and perimeter (P_1–P_4) of each smaller rectangle and record these values in Table 1 of the Project Worksheet.

Next, have each student sum the 4 areas (A_T) and then sum the 4 perimeters (P_T) and record these values in Table 1 of the Project Worksheet. Each student should check his or her answers with a calculator (optional) or have another member of the group check the answers.

Students should then compile the information from each member of the group and record it in Table 2 of his or her Project Worksheet. Students should compare A to A_T and compare P to P_T. For each student, A and A_T should be very close to if not the same number, while P and P_T should not be.

Students, working together as a group, should formulate the relationship between A and A_T ($A = A_T$) and between P and P_T ($2P = P_T$). They should record these relationships in their own words (using appropriate mathematical language) and algebraically (using precise mathematical notation) on the Project Worksheet.

Finally, each student should turn in his or her Project Worksheet, along with the rectangles, to you. A Rubric for the Project Worksheet and Quiz are provided as suggestions to help you evaluate students' skills and understandings.

Suggestions for Assessment

Embedded Assessment (Anecdotal Notes)—You could walk around the classroom and observe the students as they work independently and in groups. You could note whether or not each student is (1) actively participating in the group; (2) measuring the lengths and widths of the rectangles correctly; and (3) correctly computing the areas and perimeters.

Embedded Assessment (Group Questionnaire)—As a group, the students could complete a questionnaire designed to encourage cooperative working relationships and to help them develop an effective plan of action for working with the rectangles (see sample questionnaire on page 54).

Summative Assessment (Rubric for the Project Worksheet)—A rubric could be used to assess students' participation within a group, ability to measure sides of the rectangles accurately, comprehension of the area and perimeter formulas, and analysis of the relationships between the areas and perimeters in the activity (see sample rubric on page 55).

Summative Assessment (Quiz)—To evaluate each student's level of understanding, students could be given a quiz that (1) gives the length and width of a rectangle and asks the student to find the area and perimeter of the given rectangle as well as the sum of the areas and the sum of the perimeters for any 4 rectangles the original rectangle could be cut into; (2) gives the area and perimeter of a rectangle and asks for the sum of the areas and the sum of the perimeters for any 4 rectangles the original rectangle could be cut into; and (3) gives the sum of the areas and the sum of the perimeters for any 4 rectangles that can be put together to form a rectangle and asks for the area and perimeter of the rectangle these 4 rectangles form (see sample quiz on page 56).

Enhancing Student Learning

Links to Ideas for Progress

- Find area and perimeter of a variety of polygons by substituting given values into standard geometric formulas

Suggested Strategies/Activities

Repeat the activity using a rectangle, but ask students to cut the rectangle into any types of quadrilaterals. Have the students show/justify that the relationship between the areas still holds, but that the relationship between the perimeters does not necessarily hold. The students could then be asked to specify the conditions under which a relationship between the perimeters does hold.

Students could repeat the activity with quadrilaterals that are not rectangles. Have them show/justify that the relationship between the areas still holds, but that the relationship between the perimeters does not necessarily hold. Again, ask the students to specify the conditions under which a relationship between the perimeters does hold.

Project Worksheet

Name: ____________________ Period: __________ Date: __________

Directions: Each member of a group will complete a worksheet throughout this activity.

Formulas: $A = lw$ $P = 2(l + w)$ $P = 2l + 2w$

Table 1				
Rectangle	**Length, in inches**	**Width, in inches**	**Area, in square inches**	**Perimeter, in inches**
Original rectangle	l = ____	w = ____	A = ____	P = ____
Rectangle 1	l_1 = ____	w_1 = ____	A_1 = ____	P_1 = ____
Rectangle 2	l_2 = ____	w_2 = ____	A_2 = ____	P_2 = ____
Rectangle 3	l_3 = ____	w_3 = ____	A_3 = ____	P_3 = ____
Rectangle 4	l_4 = ____	w_4 = ____	A_4 = ____	P_4 = ____
		Sums:	A_T = ____	P_T = ____

Table 2				
Member	**Area, in square inches**	**Sum of areas, in square inches**	**Perimeter, in inches**	**Sum of perimeters, in inches**
Member 1	A = ____	A_T = ____	P = ____	P_T = ____
Member 2	A = ____	A_T = ____	P = ____	P_T = ____
Member 3	A = ____	A_T = ____	P = ____	P_T = ____
Member 4	A = ____	A_T = ____	P = ____	P_T = ____
Member 5	A = ____	A_T = ____	P = ____	P_T = ____

What relationship, if any, exists between A and A_T? ________________________________

____

If a relationship exists between A and A_T, write it algebraically (be as specific as possible): ____________

What relationship, if any, exists between P and P_T? ________________________________

____

If a relationship exists between P and P_T, write it algebraically (be as specific as possible): ____________

Group Questionnaire

Name(s)/Group: ______________________ Period: __________ Date: __________

This questionnaire is designed to be used by all the members of the group to guide planning and development of an effective plan of action and to reflect on problem-solving strategies and decision-making skills.

Directions:
1. As a group, carefully read each question.
2. Read each question and circle "Yes" or "Not quite."
3. Discuss each question in your group and write a group response.

Scoring		**Questions**	**Responses**
Yes	Not quite	Can you explain the task(s) and the key elements?	Task(s):
Yes	Not quite	Have you discussed possible plans for completing the task(s)?	Best solution:
Yes	Not quite	Can you list the steps for completing the task(s)?	Steps in process:
Yes	Not quite	Did you use an appropriate strategy for each step and sub-step?	Strategies used:
Yes	Not quite	Is this task similar to other tasks you have encountered?	Explanation:
Yes	Not quite	Did the strategies your group chose and the decisions your group made work?	Reasons:

Summative Assessment—Rubric for the Project Worksheet

Name: ______________________ Period: __________ Date: __________

Directions: Note the degree of evidence the student has demonstrated for each criterion.

	4	3	2	1	
Criteria	**Exemplary Evidence**	**Much Evidence**	**Some Evidence**	**Little Evidence**	**Score**
Working in a group **(10%)**	■ Takes the initiative to get the group working immediately ■ Performs all the steps assigned ■ On task at all times	■ Starts working when prompted by another student ■ Performs nearly all the steps assigned ■ Stays on task most of the time	■ Starts working when prompted by the teacher ■ Performs few of the steps assigned ■ Needs reminders from peers or teacher to stay on task	■ Relies on others to do the task ■ Does not perform the steps assigned ■ Does not stay on task	**S_1 =** ____
Measuring the lengths and widths of the rectangles **(20%)**	■ Finds reasonably close values for all 5 rectangles	■ Finds reasonably close values for 4 rectangles	■ Finds reasonably close values for 2 or 3 rectangles	■ Finds reasonably close values for 1 rectangle	**S_2 =** ____
Finding the areas and perimeters of the rectangles and sums for Table 1 **(50%)**	■ Finds correct values for all 12 entries	■ Finds correct values for 10 or 11 entries	■ Finds correct values for 5 to 9 entries	■ Finds correct values for 1 to 4 entries	**S_3 =** ____
Formulating the relationships between the areas and between the perimeters **(20%)**	■ Finds that $A_T = A$ ■ Finds that $P_T = 2P$	■ Finds that $A_T = A$ ■ Finds that $P_T > P$	■ Finds that $A_T = A$ and that there is no relationship between P_T and P or an incorrect relationship between P_T and P ■ Finds that $P_T = 2P$ and that there is no relationship between A_T and A or an incorrect relationship between A_T and A	■ Finds that there is no relationship or finds an incorrect relationship between A_T and A and finds that there is no relationship or finds an incorrect relationship between P_T and P	**S_4 =** ____

Total Score = $0.1(S_1) + 0.2(S_2) + 0.5(S_3) + 0.2(S_4)$ = ______

Summative Assessment—Quiz

Name: ______________________ Period: __________ Date: __________

Directions: Fill in the blanks.

Rectangle(s)	Length, in inches	Width, in inches	Area, in square inches	Perimeter, in inches
Original rectangle	$l = 20$	$w = 15$	A = _____	P = _____
Rectangles 1–4			A_T = _____	P_T = _____
Original rectangle	$l = 16$	$w = 16$	A = _____	P = _____
Rectangles 1–4			A_T = _____	P_T = _____

Rectangle(s)	Area, in square inches	Sum of areas, in square inches	Perimeter, in inches	Sum of perimeters, in inches
Original rectangle	$A = 100$		$P = 50$	
Rectangles 1–4		A_T = _____		P_T = _____
Original rectangle	$A = 80$		$P = 84$	
Rectangles 1–4		A_T = _____		P_T = _____

Rectangle(s)	Area, in square inches	Sum of areas, in square inches	Perimeter, in inches	Sum of perimeters, in inches
Original rectangle	A = _____		P = _____	
Rectangles 1–4		$A_T = 500$		$P_T = 420$
Original rectangle	A = _____		P = _____	
Rectangles 1–4		$A_T = 400$		$P_T = 160$

Total Points for Quiz __________ **/16 =** __________**%**

Instructional Activities for PLAN Mathematics

Why Are Additional Instructional Activities Included?

The set of instructional activities that begins on page 58 was developed to illustrate the link between classroom-based activities and the skills and understandings embedded in the PLAN Mathematics Test questions. The activities are provided as examples of how classroom instruction and assessment, linked with an emphasis on reasoning, can help students practice skills and understandings they will need in the classroom and in their lives beyond the classroom. It is these skills and understandings that are represented on the PLAN Mathematics Test.

A variety of thought-provoking activities, such as applying mathematics, making connections within mathematics and to other areas, small- and large-group discussions, and both independent and collaborative activities, are included to help students develop and refine their skills in many types of situations.

The instructional activities that follow have a similar organizational structure as the one in the previous section. *Like the other activity, these activities were not developed to be a ready-to-use set of instructional strategies.* ACT's main purpose is to illustrate how the skills and understandings embedded in the PLAN Mathematics Test questions can be incorporated into classroom activities.

For the purpose of this part of the guide, we have tried to paint a picture of the ways in which the activities could work in the classroom. We left room for you to envision how the activities might best work for you and your students. We recognize that as you determine how best to serve your students, you take into consideration your teaching style as well as the academic needs of your students; state, district, and school standards; and available curricular materials.

The instructional activities are not intended to drill students in skills measured by the PLAN Mathematics Test. It is never desirable for test scores or test content to become the sole focus of classroom instruction. However, considered with information from a variety of other sources, the results of standardized tests can help you identify areas of strength and weakness. The activities that follow are examples of sound educational practices and imaginative, integrated learning experiences. As part of a carefully designed instructional program, these activities may result in improved performance on the PLAN Mathematics Test—not because they show how to drill students in specific, isolated skills but because they encourage thinking and integrated learning. These activities can help because they encourage the kind of thinking processes and strategies the PLAN Mathematics Test requires.

Guiding Principles

- "Cooperative learning activities tap the social power of learning better than competitive and individualistic approaches." (Zemelman et al., 1993, p. 8)
- "Assessment is a communication process in which assessors . . . learn something about what students know and can do and in which students learn something about what assessors value." (National Council of Teachers of Mathematics [NCTM], 1995, p. 13)

Sequences

College Readiness Standards

- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor
- Apply number properties involving odd/even numbers and factors/multiples
- Manipulate expressions and equations

Description of the Instructional Activity

Students could explore special but familiar sequences (e.g., odd numbers or multiples of given numbers). They could describe the sequences in words and then write the sequences algebraically (e.g., $2n – 1$) and recursively (e.g., $a_0 = 1$; $a_n = a_{n-1} + 2$). They could investigate what happens when sequences are added together, determining the algebraic and recursive definitions of the resulting sequences.

Students, either individually or in groups, could find real-world examples of sequences (e.g., parking-ramp costs or the purchase of multiple items). These examples can be discussed and connections to other mathematical topics can be made (e.g., rate of change, slope, and functions).

Suggestions for Assessment

Pencil-and-Paper Tasks/Rating Scale—Students could be given the first few terms or a rule for several sequences and be asked to write the rule in another form or a particular term of the sequence. The teacher could ask students to give their answers for one of the sequences and explain how they arrived at these answers. The teacher could rate the students, using a scale of 1–3, on the accuracy of their work and how well they communicated the mathematics.

Multiple-Choice Questions—Students could be given a series of multiple-choice questions where they would identify another representation for a given sequence or the representation that yields a different sequence than the others.

Ideas for Progress

- Apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring
- Create expressions that model mathematical situations using combinations of symbols and numbers
- Explain, solve, and/or draw conclusions for complex problems using relationships and elementary number concepts

Suggested Strategies/Activities

Students could look at sequences pictorially or using manipulatives (e.g., triangular numbers) and then write the first few terms and the rule in various forms. The teacher could ask students to develop their own sequences and challenge others to find the 100th term or to write the rule for the sequences algebraically and/or recursively. They could explore real-world settings, identifying objects in nature exhibiting natural patterns (e.g., pineapples, pinecones, or sunflowers) and writing a rule for the pattern in various forms.

Guiding Principles

- "Powerful learning . . . comes from developing true understanding of concepts and higher order thinking associated with various fields of inquiry and self-monitoring of thinking." (Zemelman, Daniels, & Hyde, 1993, p. 8)
- "Calculators, computers, and related technology used as tools in the teaching and learning of mathematics transform the learner from calculator to critical thinker." (Lacampagne, 1993, p. 9)
- "Assessment should be a means of fostering growth toward high expectations." (NCTM, 1995, p. 1)

Curve Fitting

College Readiness Standards

- Locate points in the coordinate plane
- Interpret and use information from graphs in the coordinate plane
- Solve real-world problems using first-degree equations
- Manipulate expressions and equations

Description of the Instructional Activity

The activity focuses on the collection of data and use of data to make predictions. The data can come from many sources such as surveys, science experiments, reference books, or the Internet and should have a strong relationship (e.g., direct or inverse variation) between two variables.

Students could collect data (e.g., the winning times or distances of an Olympic event during the twentieth century or the temperature of coffee as it cools over time). The data can then be displayed in a coordinate graph and a line of best fit can be drawn. Using mathematical procedures, students could determine an equation of that line. Students could then use the equation to make predictions for several points beyond the graph.

Students could use the curve-fitting capabilities of a calculator or a computer program to obtain a linear model, comparing the equation and predictions to their own. Students could discuss the accuracy of the model and brainstorm as to why all the data points do not lie on the line and whether the predictions beyond the graph are realistic.

Suggestions for Assessment

Performance Task—Students could be given a set of data with which to determine a graph, an equation, and predictions. Students could compare their results with one or more students, possibly making revisions before turning in their final results. Students could also be asked to summarize the process they used, their results, and what they have learned. A scoring rubric can be used to assess the clarity, completeness, and logic of the mathematical skills and understandings demonstrated.

Journal—Students could include a written summary about the process they used, their results, and what they have learned. Using the predictions from their model(s), they could also discuss which model is the most appropriate one and why, how accurate they think their predictions are using that model, and why all data points do not lie on the modeling curve.

Linking Instruction and Assessment

Strands: *Data Analysis; Expressions, Equations, & Inequalities; Graphical Representations*

Ideas for Progress

- Interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems
- Gather, organize, display, and analyze data in a variety of ways to use in problem solving
- Represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)
- Create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings
- Formulate expressions, equations, and inequalities that require planning to accurately model real-world problems (e.g., direct and inverse variation)

Suggested Strategies/Activities

Using graphing calculators, computer software, and the Internet, students could explore several sets of data obtained from industrial technology, health occupations, business and marketing, and agriculture and agribusiness. They could attempt to predict if the trend in the data would tend to produce a linear or nonlinear graph. For nonlinear data, they could explore how the shape of the curve resembles different relationships such as quadratic, inverse variation, absolute value, and exponential. They might also investigate maximum and minimum values of graphs and explain how each would be interpreted. Students could generalize about appropriate models they might encounter and use in the workplace.

Guiding Principles

- "Knowledge of students' understandings and ways of thinking helps teachers to construct worthwhile mathematical tasks." (NCTM, 1991, p. 13)
- Computers and calculators can speed computation and provide rich learning experiences. Students need to learn to use that technology appropriately and responsibly.
- "[One of the] main goal(s) of assessment is to advance students' learning and inform teachers as they make instructional decisions." (NCTM, 1995, p. 13)

Estimation and Computation

College Readiness Standards

- Perform common conversions (e.g., inches to feet or hours to minutes)
- Estimate or calculate the length of a line segment based on other lengths given on a geometric figure
- Use geometric formulas when all necessary information is given
- Use relationships involving area, perimeter, and volume of geometric figures to compute another measure

Description of the Instructional Activity

Students could discuss methods or formulas for computing various measures (e.g., area, volume, or surface area). Students could then be provided with opportunities to apply these measures in real-world settings (e.g., compute amount of carpet, amount of paint, or length of a shelf). The class could discuss the precision needed, when it is appropriate to estimate, and different methods of estimating.

Students could also determine a range within which a reasonable answer can be expected to fall. Then using that range, in addition to the required precision, students could choose the appropriate method(s) of computation or estimation.

Students could be given a set number of minutes (e.g., 3 to 5 minutes) in which to identify an appropriate computation method (e.g., calculator, estimation, mental, or pencil and paper) and then use that method to determine the answer to 10–15 multiple-choice questions. The teacher could give reasonable methods and the answer for each question. As part of a class discussion, students could provide rationales supporting their choice of method.

Suggestions for Assessment

Multiple-Choice Questions—Students could be given 15–20 multiple-choice questions and a short time limit in which to identify an appropriate method of computation and to determine the answer, and justify the method used.

Journal—Students could write in their math journal, describing the different estimation techniques they practiced and providing an example of when each technique is appropriate.

Linking Instruction and Assessment

Strands: *Basic Operations & Applications; Measurement; Properties of Plane Figures*

Ideas for Progress

- Investigate and build understanding of the concept of percentage as a comparison of a part to a whole
- Do multistep computations with rational numbers
- Make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas

Suggested Strategies/Activities

Students could explore the significance of a small measurement error or an error due to the use of rounding compounded over a series of computations. They could use the Internet, newspapers, or interview members of businesses in their local community to research business and marketing situations where paychecks, sales, commissions, and compound interest might be computed. They could visit a local hospital and talk to the health professional who calculates dosages of medications or performs drug tests. They could research industrial technology environments where electrical equipment might be designed and produced. Students could discuss how a small error could ruin a family vacation or how calculations for the cost of materials needed for a home-remodeling project might affect the amount of money they need to borrow. They could brainstorm, analyze, and draw conclusions about situations or occupations in which a small measurement error, a rounding error, or an estimate would be problematic.

Linking Instruction and Assessment

Strands: *Properties of Plane Figures; Measurement; Data Analysis; Graphical Representations*

Guiding Principles

- "Active, hands-on, concrete experience is a powerful and natural form of learning." (Zemelman et al., 1993, p. 7)
- "Students learn best when faced with genuine challenges, choices, and responsibility in their own learning." (Zemelman et al., 1993, p. 8)
- "Observing, questioning, and listening are the primary sources of evidence for assessment that is continual, recursive, and integrated with instruction." (NCTM, 1995, p. 46)

Special Quadrilaterals

College Readiness Standards

- Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)
- Translate from one representation of data to another (e.g., a bar graph to a circle graph)
- Interpret and use information from graphs in the coordinate plane
- Use the distance formula

Description of the Instructional Activity

Students could be shown 5–10 examples of polygons, some that fit and some that do not fit the definition of a particular quadrilateral. Based on the examples, the students could write definitions for each type of quadrilateral and discuss them.

After the class has agreed on a definition of each quadrilateral, the class could be divided into groups and each group assigned a type of quadrilateral to investigate. Each group could make and test conjectures regarding features such as lengths of sides, measures of angles, diagonals, or bisectors, and describe how the figures are related to other quadrilaterals. As the groups are working on developing their list of characteristics and properties, students could be asked questions regarding the group's hypotheses, processes, and conclusions.

The conclusions of each group could be shared by demonstrating to the rest of the class the characteristics and properties of their assigned quadrilateral. The students could test the conclusions by developing a logical argument to support their conclusions or by finding a counterexample. Students could then organize their knowledge by comparing and contrasting the characteristics and properties of various quadrilaterals in either chart or essay format.

Suggestions for Assessment

Checklist—Each group could make a poster or a bulletin board display showing the characteristics and properties that they discovered. A diagram (e.g., tree or Venn) could also be created that shows how the various quadrilaterals are related. The teacher could use a checklist to identify whether the displays show an understanding of mathematical relationships, effectively communicate the mathematical ideas and relationships, and are of acceptable quality. Specific observations can be noted in a comment section on the checklist.

Pencil-and-Paper Tasks—Students could be given a set of statements about possible characteristics and properties and be asked to decide whether each statement is always true, sometimes true, or never true. Students could also be asked to provide a counterexample for those items that they answered sometimes true or never true.

Linking Instruction and Assessment

Strands: *Properties of Plane Figures; Measurement; Data Analysis; Graphical Representations*

Ideas for Progress

- Describe, compare, and contrast plane and solid figures using their attributes
- Gather, organize, display, and analyze data in a variety of ways to use in problem solving
- Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures
- Make generalizations, arrive at conclusions based on conditional statements, and offer solutions for new situations that involve connecting mathematics with other content areas

Suggested Strategies/Activities

Students could explore special quadrilaterals in real-world settings by identifying architectural structures in the community and objects in nature that possess these shapes. Students could then investigate perimeter and area formulas for the special quadrilaterals. They could also be challenged to find perimeters and areas of irregularly shaped regions that could be made up of several special quadrilaterals. Students could also be introduced to scale drawings by looking at the blueprints of the architectural structures they have identified from the community. They could brainstorm, discuss, and draw conclusions about why a particular shape was used (e.g., aesthetics, structural strength).

Linking Instruction and Assessment

Strands: *Graphical Representation; Probability, Statistics, & Data Analysis; Expressions, Equations & Inequalities*

Guiding Principles

- "Students learn to communicate in a variety of ways by actively relating physical materials, pictures, and diagrams to mathematical ideas, by reflecting upon and clarifying their own thinking, by relating everyday language to mathematical ideas and symbols, and by discussing mathematical ideas with peers." (Zemelman et al., 1993, p. 75)
- "Teachers who listen to students, and who plan instruction based on what they learn from listening, transform student learning." (Lacampagne, 1993, p. 4)
- "[Assessment should] elicit the use of mathematics that is important for students to know and be able to do." (NCTM, 1995, p. 58)

Representations of a Domain

College Readiness Standards

- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Read tables and graphs
- Translate from one representation of data to another (e.g., a bar graph to a circle graph)
- Manipulate expressions and equations
- Match number line graphs with solution sets of linear inequalities

Description of the Instructional Activity

Students could brainstorm about applications of domains for a variable in real-world settings (e.g., tolerance, political polls, and ranges). They could then find examples in newspapers and magazines.

Students could discuss words and phrases that indicate a domain or set of values (e.g., plus or minus, at least, no more than, or between). Students could determine which of the phrases are equivalent and when one phrase would be preferred over another. Different types of intervals and graphs representing domains or sets can also be discussed (e.g., open/closed, discrete/continuous, union/intersection, or absolute value).

Students could be given examples of domains or sets in a variety of representations (e.g., words, inequalities, interval notations, sets, or graphs). They could practice translating from one representation to the others. Students could use the examples from the newspaper and translate them to a different representation. Or, pairs of students could be given decks of "cards" that have cards with 12 or so intervals given in two different formats. The students could match representations of the same domain or set in a game format (e.g., Concentration).

Suggestions for Assessment

Performance Task—Students could find several examples of domains or sets in newspapers or magazines and show them in the various representations. This could be done as a group scavenger hunt in which each group needs to find examples of domains or sets whose representations have various characteristics (e.g., closed, discrete, conditional, or upper bound).

Anecdotal Notes/Checklists—Students could be observed while playing the game or doing the scavenger hunt. The teacher could use checklists or written notes to determine whether students are using appropriate terminology, actively participating, and demonstrating an understanding of intervals.

Linking Instruction and Assessment

Strands: *Graphical Representation; Probability, Statistics, & Data Analysis; Expressions, Equations & Inequalities*

Ideas for Progress

- Gather, organize, display, and analyze data in a variety of ways to use in problem solving
- Interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems
- Create and solve linear equations and inequalities that model real-world situations
- Represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)
- Explore and use different methods to solve systems of equations

Suggested Strategies/Activities

Domains and sets of values could be extended to two dimensions to include regions in the coordinate plane. Students could be given a region and asked to write a system of inequalities to define it. Students could also work with real-world applications as an introduction to linear programming. They could contact local businesses or agencies in the community and investigate how they employ linear programming or optimization procedures. Other ideas and lesson plans are also made available on the Internet by The Gateway to Educational Materials at http://www.thegateway.org.

Putting the Pieces Together

ACT developed this guide to show the link between the PLAN Mathematics Test results and daily classroom work. The guide serves as a resource for teachers, curriculum coordinators, and counselors by explaining what the College Readiness Standards say about students' academic progress.

The guide explains how the test questions on the PLAN Mathematics Test are related to the College Readiness Standards and describes what kinds of reasoning skills are measured. The sample instructional activities and classroom assessments suggest some approaches to take to help students develop and apply their reasoning skills.

Where Do We Go From Here?

ACT recognizes that teachers are the essential link between instruction and assessment. We are committed to providing you with assistance as you continue your efforts to provide quality instruction.

ACT is always looking for ways to improve its services. We welcome your comments and questions. Please send them to:

College Readiness Standards
Elementary and Secondary School Programs (32)
ACT
P.O. Box 168
Iowa City, IA 52243-0168

"A mind, stretched to a new idea, never goes back to its original dimensions."

— Oliver Wendell Holmes

What Other ACT Products and Services Are Available?

In addition to the College Readiness Standards materials, ACT offers many products and services that support school counselors, students and their parents, and others. Here are some of these additional resources:

ACT's Website—**www.act.org** contains a host of information and resources for parents, teachers, and others. Students can visit **www.planstudent.org**, which is designed to aid students as they prepare for their next level of learning.

The ACT—a guidance, placement, and admissions program that helps students prepare for the transition to postsecondary education while providing a measure of high school outcomes for college-bound students.

EXPLORE—an eighth- and ninth-grade assessment program designed to stimulate career explorations and facilitate high school planning.

WorkKeys®—a system linking workplace skill areas to instructional support and specific requirements of occupations.

ACT Online Prep™—an online test preparation program that provides students with real ACT tests and an interactive learning experience.

The Real ACT Prep Guide—the official print guide to the ACT, containing three practice ACTs.

DISCOVER®—a computer-based career planning system that helps users assess their interests, abilities, experiences, and values, and provides instant results for use in investigating educational and occupational options.

Bibliography

This bibliography is divided into three sections. The first section lists the sources used in describing the PLAN Program, the College Readiness Standards for the PLAN Mathematics Test, and ACT's philosophy regarding educational testing. The second section, which lists the sources used to develop the instructional activities and assessments, provides suggestions for further reading in the areas of thinking and reasoning, learning theory, and best practice. The third section provides a list of resources suggested by classroom teachers.

(Please note that in 1996 the corporate name "The American College Testing Program" was changed to "ACT.")

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